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МОНОЛОГИ, ДИАЛОГИ, ДИСКУССИИ

ОТ РЕДКОЛЕГИИ: В статье Марко Негри «Воображающая философия» (Negri M. Imagining philosophy. – Вестник Томского государственного университета. Философия. Социология. Политология. – № 2(14), 2011, С. 182–200) предпринимается попытка подвергнуть сомнению утверждение, что философия является сугубо рациональной деятельностью. В статье «Воображающая логика», являющейся продолжением, предпринята попытка представить логику как работу с представлениями, но не с понятиями.

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Marco Negri

IMAGING LOGIC

Logic rests on forms or structures. Logical arguments or inferences - in which one passes from a true premise to a true conclusion - depend on a person's capacity to put into focus the relation between the image expressed by the premise and the image expressed by the conclusion. There are, it is maintained here, two distinct iconic models at the basis of a logical argument or inference: i) a containment model, according to which one passes from a true premise to a true conclusion because the image suggested by the conclusion is necessarily contained in the image suggested by the premise (e.g. 'If I am in a house then I am in a house'); and ii) an entailment model, according to which one passes from a true premise to a true conclusion because the image suggested by the conclusion necessarily crosses or permeates the image suggested by the premise (e.g. 'If I am in a house then I am in space'). By explicitly drawing and observing the logical form or structure at the basis of a given reasoning one could thus prove or disprove its correctness - one could, immediately or more immediately, demonstrate its validity or invalidity. A 'pointography' (or 'dottography') is a way of describing logical situations by means of points/dots: it is a way of representing logical situations that crucially exploits the simplest elements of images. A 'pointography' (or 'dottography') is an adequate device for showing the minimal form or structure of an argument or inference, though it is in the end only the eye or mind eye that makes it possible for one to experience such form or structure – this is the reason why it is the eye or mind eye's axiom that one should put at the very beginning of logic.

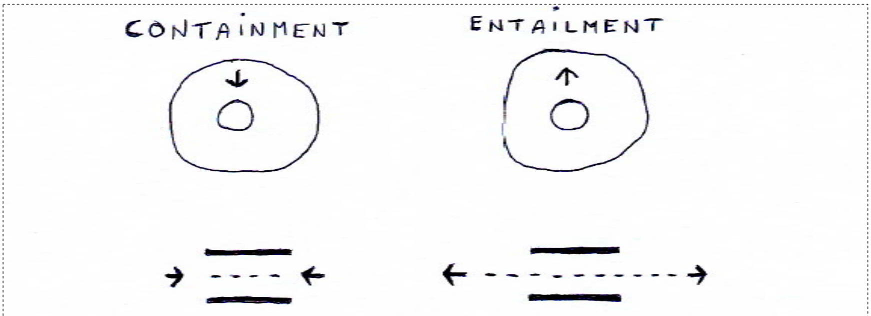
Keywords: *images, drawing, pictorial forms or structures of i) containment and ii) entailment, 'pointography' ('dottography'), mind eye's axiom, logical vision.*

Introduction

Could we see logic? Which are some of the most relevant images in logic? What do we see when we understand logic? (Is it not perhaps true that when we understand logic we somehow see it?) Is it possible to make logic more evident? Logic is crucially concerned with correct reasonings or arguments, that is, with ways of passing from true premises (or at least valid premises) to true conclusions (or at least valid conclusions). So, when are these reasonings or arguments correct? When do these reasonings or arguments work? They work all the times in which a subject of experiences is able to focus, and thus reflect, on different kinds of forms, or structures (or models, or patterns, etc.). If one is able phenomenologically to focus and describe these forms or structures, then one has also the opportunity to *show* these forms or structures to other persons: one can detect and display the relations among the elements of these forms or structures. A real proof in logic has thus always the features of a literal *demonstration*: a reasoning that one could in principle always reconstruct visually. Logic, it is maintained here, is not very different in spirit from geometry, though logic deals with more abstract ideas than geometry (the importance of geometry for philosophy was wholly recognized by Plato). Since the ideas of logic are abstract it is of the utmost importance to draw such ideas. On the contrary, strict symbolic logic, that is, mathematical logic in the style of written algebra, has never faced the problem of making its notations, and thus its representations, more impressionistic and explicit. But one cannot precisely grasp the meaning of a logical symbol if one is not able to say which images are generated by the use of such symbol (and thus which images could fall under such symbol). For example: one is not able to grasp the meaning of '⌐ A' if one does not know that it generates a figure of negation ('⌐' means 'not'): a figure in which something (or someone) A is imagined to be absent, or cancelled, and thus it is imagined to designate only, at most, what is external with respect to A itself, etc. Now, if one examines the first developed logic of the ancient times, the logic of Aristotle, one sees it is based on certain basic figures too: the four famous Aristotelian syllogistic figures 'A, E, O, I', with each of such letters referring to a paradigmatic structure of categorical propositions (here, precisely, the letter 'A' designates a structure of 'universal affirmatives' ('All X are Y'); the letter 'E' designates a structure of 'universal negatives' ('No X are Y'); the letter 'O' designates a structure of 'particulars affirmatives' ('Some X are Y'); and finally, the letter 'I' designates a structure of 'particular negatives' ('Some X are not Y')). Figures, in short, are crucial for Aristotle's logic too. However: is Aristotle's logic wholly coherent with a philosophy centered on images? No, at least not if one takes into account the following decisive fact: that the four main figures of Aristotle's logic are not built by analyzing different perceivable images, but by analyzing different linguistic terms and sentences (this fact about Aristotle's logic is for example lucidly stressed by Descartes in his '*Replies*', in particular when he observes that his 'cogito ergo sum' argument has not the features of a terminological syllogism but of a belief grounded on an immediate act of mental intuition or vision). Aristotle's idea of developing logic on a strict linguistic basis is then embraced, more than twenty centuries later, by Gottlob Frege (indeed Frege's prominence in logic is due, among the other things, to his ability to extend Aristotle's term logic – this by conceiving what is nowadays called predicate or quantificational logic). Frege's main device for representing some basic logical notions (identity, implication, etc.) is the *Begriffsschrift*, a German expression to be translated as 'concept writing' or 'concept script': a notational system modeled on formulas used in algebra and arithmetic, to be applied to linguistic propositions. In Frege, once again, one thus finds this: that logic is not conceived as a way of shading light on perceivable forms or structures at the basis of our valid reasonings, but as a way of packing words and sentences into formulas. In short: first Aristotle and then, above all, Frege somehow set the path that brings logic to its present indirect shape and symbolization (the attempt by the American logician Charles Sanders Peirce to make logic more explicit and iconic is not in the end successful for Peirce does not anchor his logic on a robust philosophy of images – he anchors it on a mere semiotic or 'semeiotic' reflection, a mere philosophy of signs).

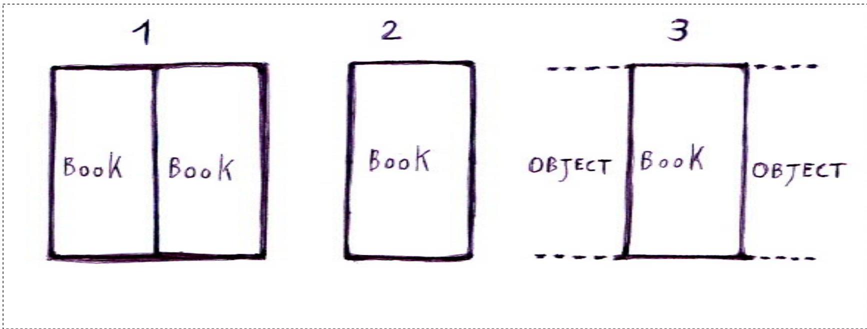
But one could now ask: would not it be possible to invert the main trend in logic that wants it more and more departed from our impressions and ideas? Would not it be possible to reconcile logic with our experiences, so to make logic closer, to begin, to our visual impressions and ideas? It is possible: it is possible if one realizes that one's activity of reflexive imagination plays an indispensable role in logic; such activity, however, should then also be accompanied by one's capacity *publicly* to reorganize and display one's internal images.

One could begin here by noticing this: that if one depicts the most important logical reasonings, one sees that they take two basic iconic forms: i) a form or structure to be described as (literal) *containment* – as, for example, with the inference 'If I have two books then I have one book'; and ii) a form or structure to be described as (literal) *entailment* (or *implicature* or *involvement*) - as, for example, with the inference 'If I have a book then I have an object' (while an inference as 'If I have a book then I have a blue object' would be incorrect). Logical reasonings that depend on the form of i) containment are inferences that start from something bigger (in this sense external) and move towards something smaller (in this sense internal). Logical reasonings that depend on the form of ii) entailment (in the strict sense) are inferences that start from something smaller (in this sense internal) and move towards something bigger (in this sense external).



Cases of logical inference expressing the very same idea of identity ('a = a', 'a = b', etc.) could then also be seen to point to boundary forms or structures of containment: they are an extreme variant of the containment kind (one central idea of logical identity is the idea of something '(being) *in* itself'). Indeed, the basic form of these 'identity inferences' is that of a geometric figure containing itself or another geometrically congruent figure – as, for example, with the inference 'If I have a book then I have a book'.

(To recapitulate about the previous examples: as for the inference (1) 'If I have two books then I have one book' one sees this: that the image of two books necessarily contains, as its own part, the image of one book. As for the inference (2) 'If I have a book then I have a book' one sees this: that the image of a book necessarily contains itself, that is, a congruent image of the book. As for the inference (3) 'If I have a book then I have an object' one sees this: that the image of a book necessarily entails or involves (or ingenerates, etc.) the image of an object.)



The form of i) containment makes then possible to see the most intuitive inferences associated with the so called propositional logical connectives: negation ('If *not a* then *logical absence of a*'); conjunction ('If *a and b* then *a*, *b*'); (inclusive) disjunction ('If *a or b* then *a*, *b*'. Etc.); conditional ('*If a then b* then *a*, *b*'. Etc.); biconditional ('*If and only if a then b* then *a*, *b*'. Etc.). In these cases, one is able to evaluate the correct inferences also because one is able to see the specific images that are associated with each specific logical connective: one is able to see that the image of negation of one thing contains the image of absence of such thing (negation); one is able to see that the image of two things together contains the image of each one of such things (conjunction). Etc.

What about the relevance of the form of ii) entailment for reasonings exploiting the five propositional connectives? One could reflect here on an inference like this: 'If *a*, *b* then *a and b*'. In this case, if one accepts this inference as correct, one does so because one sees that *compositionally* the image of 'one thing, another thing' brings about, and thus entails, the image of 'one thing *and* another thing' (not surprisingly, the truth-tables for the logical connectives have historically been built, at the beginning of the twentieth century, in a compositional way: they rely on a philosophy of logical atomism).

The forms or structures of i) containment and ii) entailment are also at the basis of the central reasonings in what is called 'predicate logic' or 'quantificational logic'. One could begin to consider, here, certain inferences in the style of Aristotle: if one sees, for example, that 'If all books are interesting then some books are interesting' one just observes that the image of 'all' (expression of a universal quantifier) contains the image of 'some' (expression of an existential quantifier). Similarly: if one sees that 'If no books are interesting then some books are not interesting' one just observes that the image of 'no one thing' (expression of a negation plus an existential quantifier) contains the image of 'something does not' (expression of an existential quantifier plus a negation). Moreover: the rule of inference of universal instantiation, that allows one to *eliminate* the universal quantifier in a logical demonstration, depends on the form or structure of containment: 'If everything is a book then one concrete thing is a book'. The rule of inference of universal generalization, that allows one to *introduce* the universal quantifier in a logical demonstration, depends instead on the form or structure of entailment: 'If every concrete thing is a book then all things are a book'. Similarly: the rule of inference of existential instantiation, that allows one to *eliminate* the existential quantifier in a logical demonstration, depends on the form or structure of containment: 'If something is a book then a concrete thing is a book'. The rule of inference of existential generalization, that allows one to *introduce* the existential quantifier, depends instead on the form or structure of entailment: 'If a concrete thing is a book then something is a book'. (In other terms, about these rules of inference in quantificational logic: one sees that a bound variable contains all the constants in the domain of the bound variable; and one sees that all the constants in the domain of the bound variable entail the bound variable.)

Let us consider now the so called 'modal logic', the logic built on the ideas of necessity and possibility. One could observe here, for example, that the inference 'If it is necessary that I have a book then I have a book' rests on a form or structure of containment (one here imagines this: if the logical space must host a book then it hosts a book). Or one could observe, for example, that the inference 'If I have a book then it is possible that I have a book' rests on a form or structure of entailment (one here imagines this: if there is a book in the logical space then there is available logical space for the book in the logical space). Or one could observe, for example, that the inference 'If it is possible that I have a book then it is not necessary that I have a book' rests on a form or structure of containment (one here imagines this: if there is further available space in the logical space that hosts a book then such further space is not null). Again: one could observe, for example, that 'If it is necessary that I have a book then it is necessary that it is possible that I have a book' rests on a form or structure of entailment (one here imagines this: if the logical space must host a book then it must be possible for the logical space to host a book). Etcetera.

The iconic forms or structures of i) containment and ii) entailment are thus at the centre of logic - if one scrutinizes the most relevant images behind our reasonings. A further question is now this: is there also a way of more directly or intuitively showing – that is, demonstrating – such relevant images behind our reasonings? Yes, a possibility is this: one should here simply begin to substitute the linguistic, alphabetical letters used to represent propositions or sentences in standard symbolic logic – i.e., 'p', 'q', etc. – with geometrical *points* or *dots* referring to visualizable things: '•' (one should keep in mind here that a point/dot is the smallest image - a point/dot is the highest expression of iconic parsimony in the sense of Ockham's razor). So, for example, one could represent the image 'The book is heavy' with two points: 'Book', as subject/object: '•'; 'Being heavy', as property: '•'. For greater simplicity, one could represent a subject/object with its property (as in a subject-predicate sentence) by means of a single point (thus, 'The book is heavy' will be pictured like this: '•'). If the idea or image of a (true) 'something' is this, '•', then the idea or image of negation (the negation of such something) will be this: '¬ •'; the conjunction of two things will be this: '• & •' (or '• ∧ •'); the disjunction will be this: '• ¬ •'; the conditional: '• → •' (as containment: '• C •' (thus with • ≥ •); as entailment '• ⊃ •' (thus with • < •)); the biconditional: '• ↔ •'.

Etcetera. In order to name this technique for representing things by means of visual points (or dots) I shall coin here the term 'pointography' (or 'dottography').

One could notice that a pointography (in the sense just specified) is, among the other things, a more genuine instance of ‘ideography’ than Frege’s *Begriffsschrift* for it is sensitive to the nature of the ‘idea’ (the Ancient Greek ‘ἰδέα’) as something visual. By representing things as points/dots, and not (or not immediately) as symbolic letters or numbers, one is able to pass from verbal, indirect proofs to visual proofs - that is, to explicit demonstrations. Let us consider, for example, the following statement: ‘It is not the case that a book is blue and a book is red, and that a book is not red’. If one draws such statement and simplifies it one sees this:

¬((•&•)&¬•)
¬((•)&¬•)
¬(•&¬•)
¬(¬•)
¬¬•
•

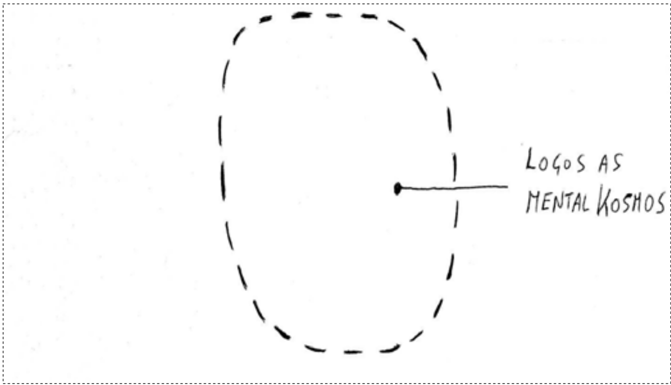
(Here thus one sees an image of conjunction, ‘a book is blue and a book is red’: ‘(•&•)’; and then an ‘image’ of contradiction, ‘a book is red and a book is not red’: ‘(•&¬•)’; then one sees an image of negation of the contradiction, thus an image of negation of something false: ‘¬(¬•)’; finally one sees an image of something true: ‘•’.) By reconstructing a certain logical situation by means of drawn points one makes it possible both for the eye and the mind’s eye more realistically to observe such situation and calmly evaluate it (this of course makes it also possible for other people publicly to observe one’s reasonings; again, it allows one to make a visible comparison of different logical situations; etc.). An important consideration to be highlighted here is however this: the possibility of arriving at a more iconic and even geometric representation of a logical state of affairs by employing a pointography (i.e. a dottography) should not be taken to imply the possibility of testing the truth or falsehood of such state of affairs without the mind eye. The belief that one could evaluate a logical case without intuition and philosophical reflection is just illusion - as Gödel has shown, the idea that one could close logic under a *finite* list of axioms, however long such finite list of axioms is taken to be, is mere chimera. Hence there is perhaps just one axiom that one could somehow put at the beginning of logic, what one could call the ‘mind eye’s axiom’: logic should be evaluated, first of all, on the basis of insight and philosophical speculation. Indeed: the ‘mind eye’s axiom’ would clearly state that a logical system cannot but be an open system. This suggests, among the other things, that one should be open to see whether there are other possibilities of interpreting a given logical case – one should be ready to take into consideration counterfactuals (i.e. counterexamples). It is also in this particular sense that the mind eye’s axiom keeps the logical system open. On the other hand, the fact that one wants logic to be based on direct, reliable evidence - as, for example, when one is dealing with visual demonstrations – suggests that one thinks that logic should be put on a firm ground. The idea of a logical space, imagined as a visualizable space, and of something positive contained in it looks as a firm, reliable ground – this is so because one just realizes here that if one is in front of a positive thing then one cannot be in front of its opposite, that is, the negation of such positive thing (as we have said, the absence of such positive thing, etc.). And one’s ability, in general, clearly to see the logical space and all the non contradictory possibilities contained in it is just one's capacity of logical vision.

1

Logic, from the written and spoken Greek word 'logos'. It has been translated as 'word', 'speech', 'thought', 'reasoning', 'action' ('deed'), etc.

Perhaps the best translation of 'logos', as conceived in Greek early philosophy, is 'mental kosmos'. One should see here that 'logos' (or 'Logos') is associated with the Greek verb 'legein': 'to recollect' (this is the reason why 'logos' could be taken to refer to the idea of 'mental kosmos' or 'mental all' – one should also notice here that the Greek word 'kosmos' refers to some intuition of 'ordered universe').

Here is a possible picture of *logos* as mental *kosmos*: (Image 1)



2

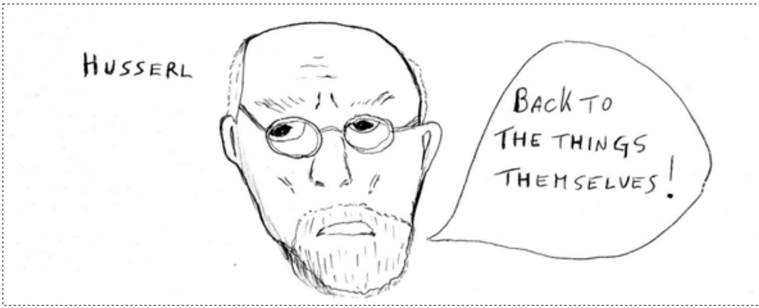
The idea of mental cosmos suggests this: that one cannot do logic - one cannot understand logic - without a theory of mind. More in general: one cannot do logic without a metaphysics.

3

Before Aristotle, logic - as discourse about *logos* – is conceived as a reflection on human beings' experiences about the principles of thought and world: before Aristotle, logic is experiential, not linguistic – with Aristotle it becomes a 'term logic'. (Aristotle's syllogistic logic is thus not built on simple intuitions. It is based, for example, on two premises and a conclusion, not on a single premise and a conclusion. Etc.)

4

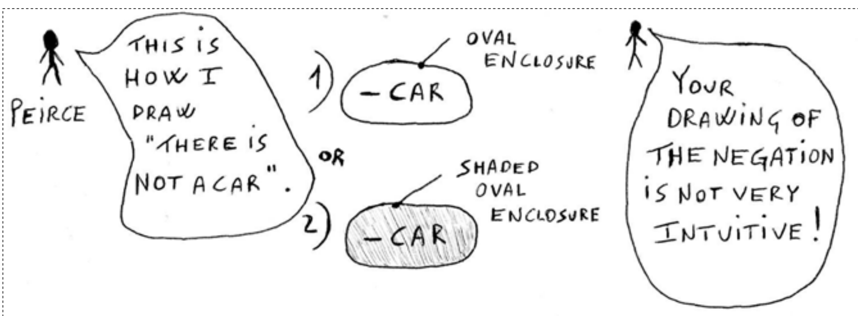
At the end of the 19th century-beginning of the 20th's, the mathematician and philosopher Edmund Husserl tries to move back to a conception of logic as a general meditation on the principles of (immediate) phenomenal thought and experience. (Image 4)



5

At the end of the 19th century-beginning of the 20th's, Charles Sanders Peirce tries to develop a more iconic approach to logic. Peirce's more iconic approach to logic is *not*, however, based on a substantive interest on images. Peirce's main idea is *not* that of putting images at the basis of his philosophical reflections, but that of putting *signs* at the basis of his philosophical reflections. This is the reason why Peirce is nowadays recognized, correctly, as one of the fathers of semiotics (or 'semeiotic'), not as a contributor to the development of a philosophy of images. Logic is conceived by Peirce as a formal branch of his pragmatic (or 'pragmaticist') theory of signs. Moreover, his writings on icons and logic are wholly based on *non* ordinary images: they are based on diagrams and graphs, specifically on 'existential graphs'. Since Peirce does not pay much attention to human beings' common experiences, many of his existential graphs come out to be counterintuitive - in his texts concerned with the existential graphs, Peirce has thus often to add technical 'conventions', or 'permissions', or assumptions, etc., in order to explain such graphs. Here is an example: when Peirce depicts the idea of negation, he counterintuitively puts a circle, i.e. an oval enclosure, around a propositional letter. For instance: 'There is not a car' is drawn as '– car' inside an oval enclosure, or a shaded oval enclosure (the sign '–', a dash, is used by Peirce to signal an assertion).

Peirce does not discuss, then, other possible ways in which people usually imagine or visualize a negation. (About this point one could also reflect on J.F. Sowa's commentaries on Peirce's manuscripts.) (Image 5)



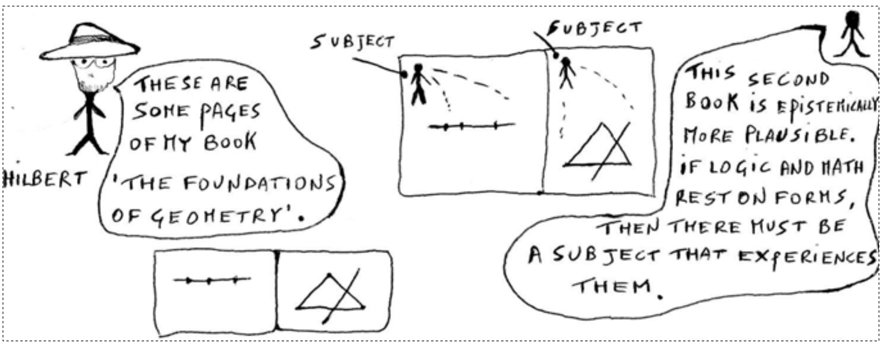
6

Nikolaj Vasilev's (in Russian, Николай Александрович Васильев's) 'imaginary logic' is not an iconic logic – it is, historically, one among the very first instances of non classic or (so called) 'paraconsistent' logic.

7

What is erroneous in David Hilbert's 'formalistic' approach to logic is this: to think that the main problem of logic is syntactically to derive or prove theorems from self-evident axioms. The main, or primary, problem is instead this: to *show* the self-evident form of the axioms or first principles. Hilbert's attempt to fix the axioms is then vitiated by the fact that his symbols and representations concerning logic and mathematics are incommensurable: they are not homogenous (they sometimes are alphanumerical symbols; sometimes geometrical elements; etc.). Moreover, Hilbert does not make it explicit that someone – a subject of experience – has to see or intuit such forms or representations (Hilbert's main idea is just that of closing a logical or mathematical object by bringing it under a finite system).

From a more general point of view, the limit of Hilbert's approach to logic and mathematics is the following: trying to be objective on logic and mathematics just by ruling on the connection of certain possible objects of logic and mathematics (e.g. finitary numerals such as 1 ('1'), 11 ('2'), 111 ('3') and so on, or Euclidean elements, etc.), without paying much attention to the conscious subjects of logic and mathematics. In one of his first works, *Foundations of Geometry*, Hilbert never presents a geometrical figure together with an explicit image of a subject perceiving or intuiting such figure. Indeed, it is only in the latter years of his studies that Hilbert begins to take into consideration representational and heuristic problems at the basis of logic and mathematics). (Image 7)



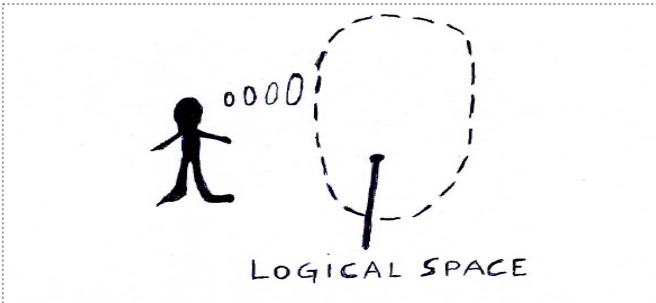
8

Intuitionistic and constructivist logics – like those developed by L. E. J. Brouwer – could be seen as attempts to put logic closer to a person's experiences. However: even these conceptions of logic have not culpably been developed on iconic bases. (Brouwer also maintains that time is the most primitive and crucial element for logic. In accordance with what I have till now tried to show, I think it is however space, that is space-time, in particular logical or experiential space-time, to be the most primitive and crucial element for logic.)

9

The space of logic, or logical space, is the space of the conceivable – one could more precisely say it is the space of the conceivable as experienceable.

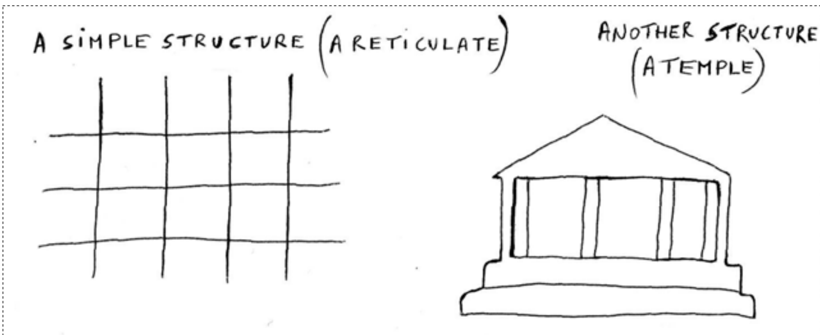
A possible drawing of such logical space is this (an open circle or an open sphere): (Image 9)



10

Logic is concerned with basic structures or forms or models (even dynamical structures or forms or models) inside the logical space.

An image of a structure is, for example, the image of a simple net or reticulate – one could then also think about such things as the structure of a temple, etc.: (Image 10)

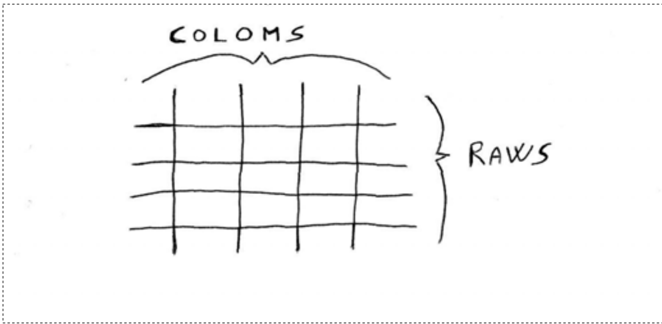


11

Developing logic: trying to show the simplest structures or forms underlying this or that phenomenon or thing (for example a truth-preserving reasoning).

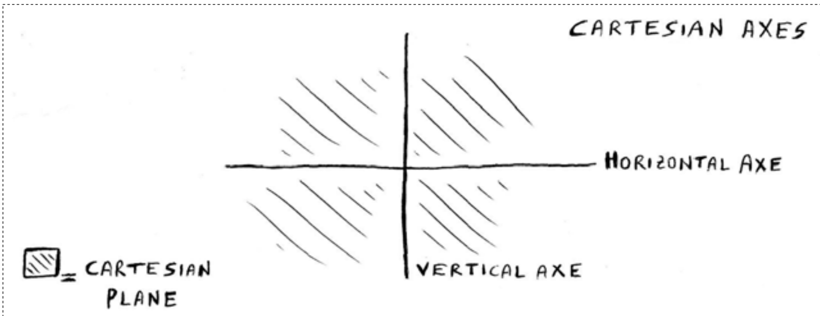
12

A net or reticulate is made, for example, of vertical and horizontal lines - that is, of columns and rows. (Image 12)



13

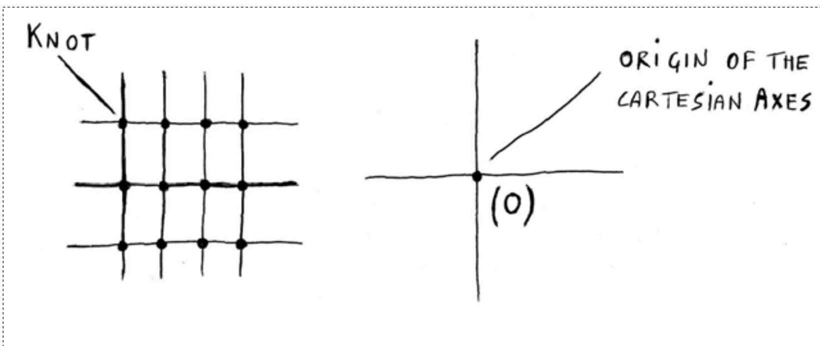
The Cartesian axes, which give rise to a Cartesian plane, are a kind of structure – here one could just observe the two perpendicular lines, without immediately paying attention to the fact that such lines could then also be ordered, etcetera. (Image 13)



14

Two perpendicular (or quasi perpendicular) lines meet in a point. This point – in particular if one has in mind a net, or a mesh, etc. – is sometimes called ‘knot’.

(The meeting-point of two Cartesian axes is usually called ‘origin’.) (Image 14)



15

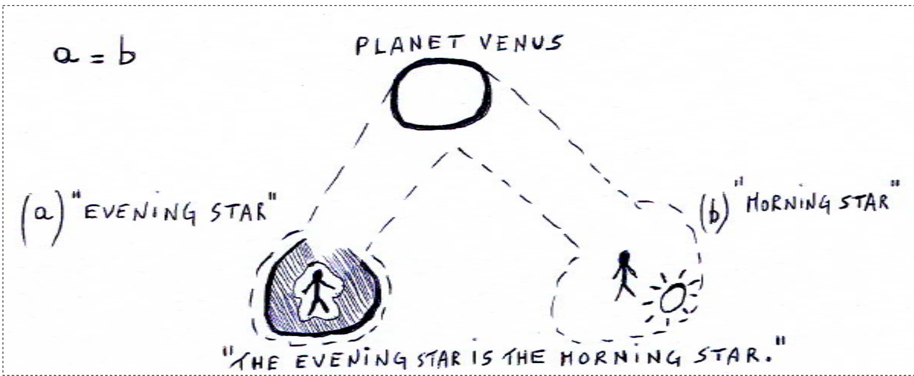
The main principle of logic is the principle of identity, the idea of something or someone being itself or oneself. Here one usually writes this: a = a.

The idea of identity is the most extreme idea of relation – it is an internal, or self-reflexive, relation. (Image 15)

16

Again on the idea of identity, in particular when it is represented as ‘a = b’: Gottlob Frege, for example, tries to explain the case in which two different thoughts point to the same object. Frege discusses the case of the ‘Evening Star’ (in German ‘*der Abendstern*’) and of the ‘Morning Star’ (in German ‘*der Morgenstern*’). Frege claims that the ‘sense’ (in German ‘*Sinn*’) ‘Evening Star’ has the same ‘reference’ (in German ‘*Bedeutung*’) of the ‘sense’ ‘Morning Star’ for both such ‘senses’ point to one object: the planet Venus.

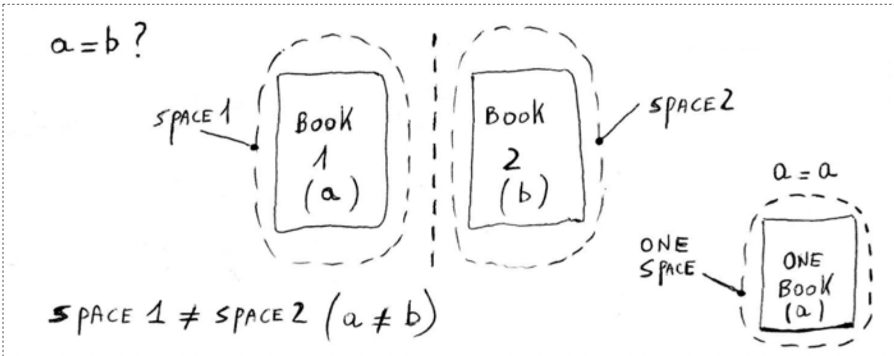
Let’s now however observe this: that we could *draw* the fact that ‘Hesperus is Phosphorus’ (‘The Evening Star is the Morning Star’) in the following way: (Image 16)



‘Hesperus is Phosphorus’ should thus not be regarded as a famous sentence of a philosophy based on language but, first of all, as a famous imaginational experience - the ‘sense’ of a thought should not be regarded first of all as a linguistic ‘mode of presentation’, but instead as a *mode or circumstance of vision* (as a visual interpretation - an interpretation that also relies, for example, upon the perception of a given environment or context or background).

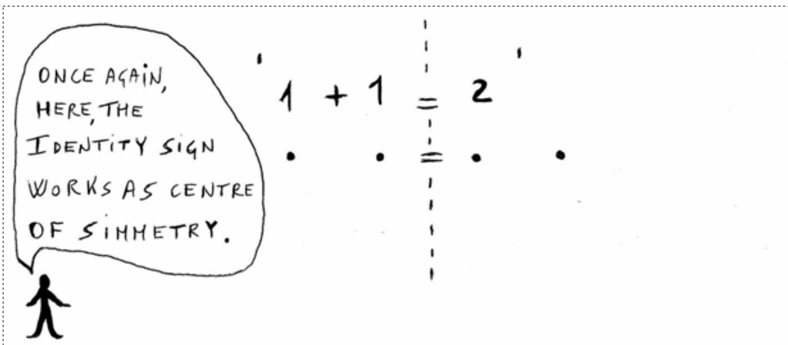
17

On the idea of identity written as ‘a = a’ or as ‘a = b’: according to Leibniz’s principle of the ‘indiscernibility of identicals’ two things are identical if they have all their properties in common. Hence, two copies of the same book are not one and the same, for they display at least one different property: they occupy two different portions of space. Of course the two copies of the same book should be seen to share the same form - or experienced form (Image 17).



18

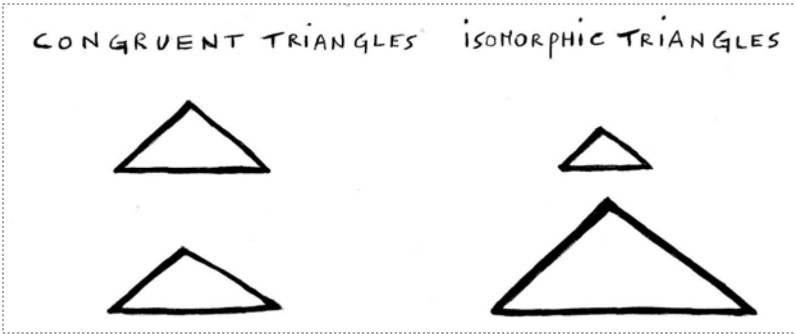
One could see that certain numbers, or certain numerical expressions, etc. have the same logical, abstract form. For example: the numeric expression ‘1 + 1’ has the same logical form as the numeric expression ‘2’ (indeed: ‘1 + 1 = 2’). One could clearly observe this if one draws the numbers or the numeric expressions as points: (Image 18)



(The points ‘• • = • •’ have different spatial positions, though one is assuming here that numbers could be held to be abstract entities – entities that (taken singularly) do not have to be characterized as displaying a contingent position in space and time.)

19

Geometric congruence is two figures having the same i) shape and ii) size: two figures are intuitively congruent if and only if the distance between two points in the first figure is the same as the distance of the two corresponding points in the second figure (geometric congruence is analogous to equality or equivalence for numbers). Isomorphism is two figures having the same shape. Geometric congruence and isomorphism could be seen to express degrees of identity. (Image 19)

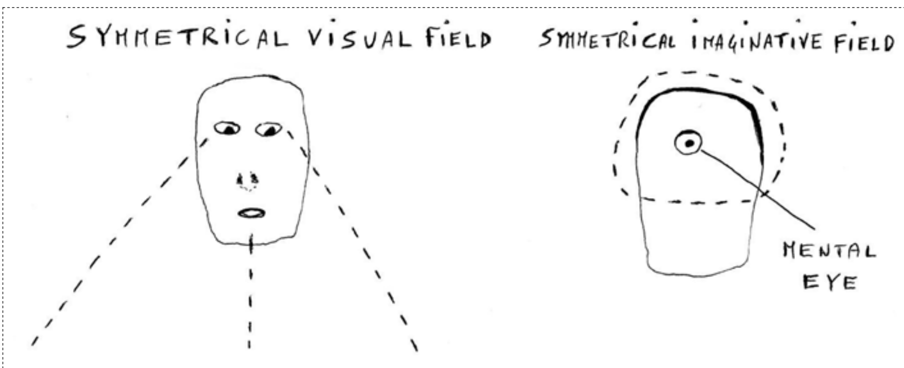


20

We have seen that the main condition of identity, conceived in the logical sense, is the condition of isomorphism, and even more strictly of congruence. One could observe such isomorphism or congruence even by paying attention to the form or shape of certain words, as, for example, in the following linguistic proposition (i.e. a tautology): '*water* is *water*' (that is, '*water* = *water*').

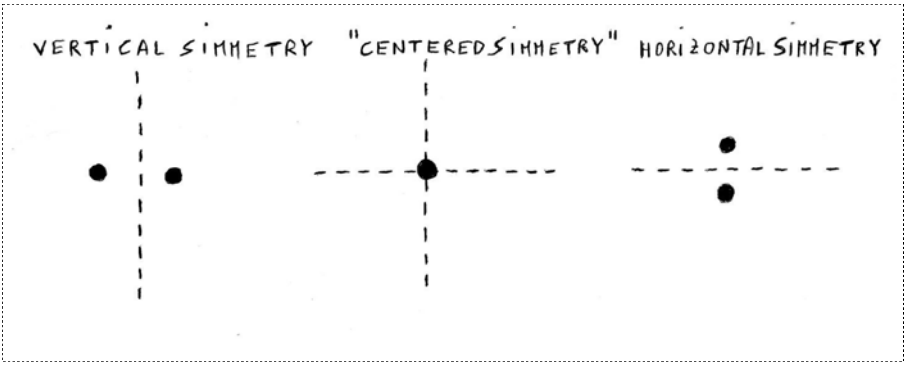
21

A relevant idea for logic is symmetry. When one is concerned with logic one should remind oneself that one's bodily eyes are symmetrical and that one's inner or mental eye is symmetrical too - one should remind oneself that one's visual *field* and imaginative *field* appear symmetrical. (Image 21)



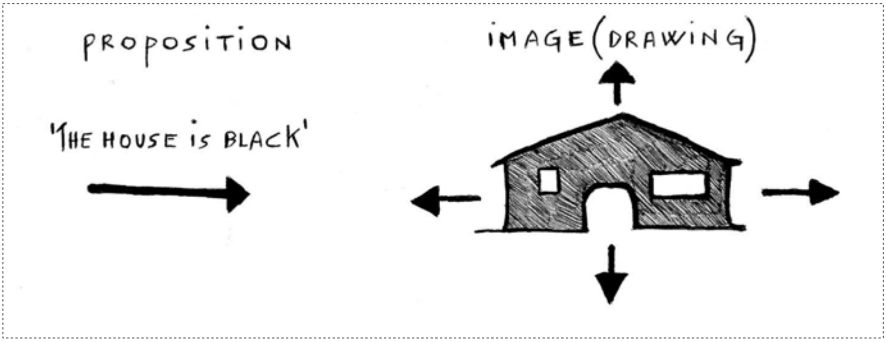
22

Here are some images of i) vertical symmetry, ii) horizontal symmetry, and iii) centered symmetry (of course the expression 'centered symmetry' should not be taken to mean that only centered symmetries have a centre – indeed every symmetry implies, just *qua* symmetry, a centre.) (Image 22)



23

One of the first difficulties in seeing some logical forms inside our ordinary or common phrases – written propositions or statements – is this: the fact that many written languages are *a*symmetrical: they are written, for example, from left to right, etc. For instance: this proposition, ‘the house is black’, is asymmetrically written from left to right. (Image 23)



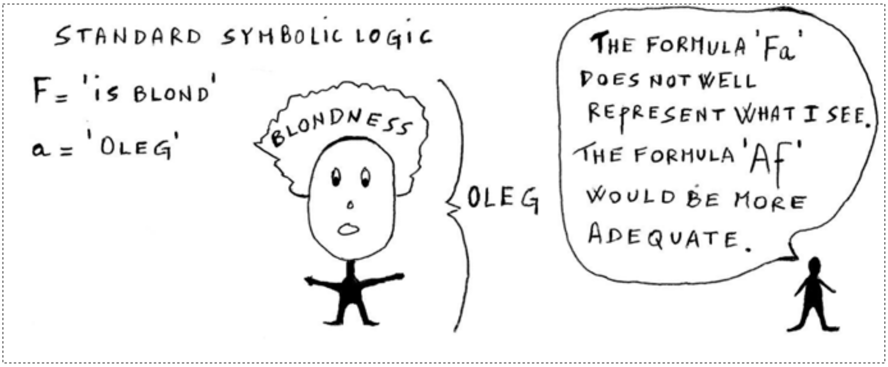
24

Symbolic logic – i.e. the development of symbolical notations in the history of logic (in particular in the 19th and 20th centuries) – has not faced the problem of making its signs and representations more impressionistic. In other words, symbolic logic has not faced the problem of making its representational conventions more natural - that is, closer to our everyday experience of the world.

(Some cases and commentaries in support of this latter claim are offered below).

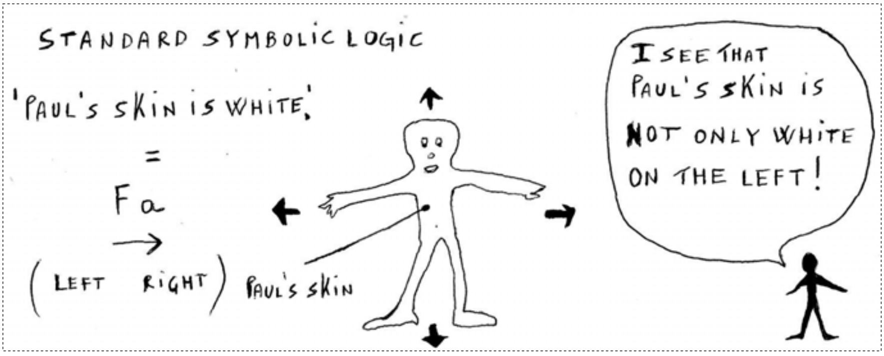
25

Classic symbolic notation has for example ‘Fa’ to refer to a certain subject or object ‘a’ that has the property or relational property ‘F’ (‘F’ stands for ‘function’). Here one immediately sees this: that the letter ‘F’ is written as a big (capital) letter and the letter ‘a’ is written as a small (low) letter. As a matter of fact, however, it should be the other way round: if it is true that the letter ‘a’ represents a subject or object and the letter ‘F’ a property, then it is the letter ‘a’ that should be written as a big letter and it is the letter ‘F’ that should be written as a small letter. Moreover: assuming a left-right order of writing, the letter ‘a’ should have precedence with respect to the letter ‘F’, and thus one should find the letter ‘a’ on the left and the letter ‘F’ on the right (indeed, in our common, non-technical way of thinking and writing, the subject is usually, realistically, put on the left, at the very beginning of the phrase). An impressionistic, or more impressionistic, notation should thus have this: ‘Af’ (for argument’s sake I am here of course making *tabula rasa* of certain conventions in symbolic logic). (Image 25)



26

When one writes the subject-predicate as ‘Fa’ one should also keep in mind this: that the predicate (or the function, etc.) ‘F’ is not, in reality, on the left of the subject ‘a’. For example, if ‘F’ stands for the property ‘being white’ and ‘a’ stands for the subject ‘Paul’s skin’, then it is not of course the case that Paul’s skin is white on the left! If a property belongs to a subject, one has to assume that this property is symmetrically distributed in the subject. (Image 26)



27

In the following points (from point 28 to point 36) I will try to put into focus the ideas of i) subject and predicate; ii) relation; iii) negation; iv) conjunction; v) disjunction; vi) material implication (conditional); vii) strictly logical implication (biconditional).

28

Classic logic is seen to be based on the ideas of subject and predicate. One could draw a logical figure of a subject and its property as follows:

-•-

(For example: if the idea is that ‘Socrates is mortal’, then the point ‘•’ draws the subject ‘Socrates’; and the little line ‘**--**’ draws the property ‘being mortal’.)

For reasons of simplicity one could draw a subject with a given property just as a single point:

•

(This latter solution might especially suit those philosophers that think that the distinction object/property should be abandoned.)

29

An image that seems to capture the idea of a (basic) logical relation could be the following: a line or segment connecting two subjects, or two objects, or two individuals, etc. (for example; if the idea is that ‘Romeo and Juliet love each other’ then one point ‘•’ will refer to the subject or individual ‘Romeo’; the other point ‘•’ will refer to the subject or individual ‘Juliet’; the line or segment connecting them ‘**---**‘ will picture their relation of love.)

Here is a possible drawing of a basic logical relation:

•---•

In some cases, one could also find useful to show the direction of a certain relation. If ‘Don Quixote loves (in an uncorresponded way) Lady Dulcinea’, one could, for example, draw this (asymmetric relation):

Don Quixote **•---•** Dulcinea

‘To love’

30

The idea of negation is usually expressed by the word ‘not’.

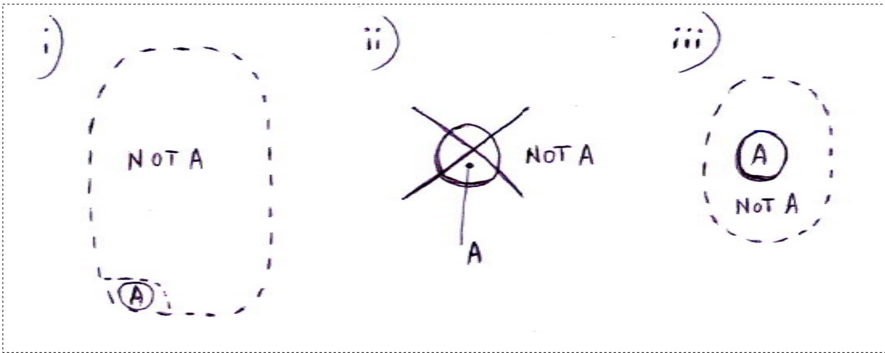
A possible image that refers to the negation of a certain thing is an image of *absence* of such thing: not A is the absence of A (here one somehow keeps A in the ‘corner’ of one’s mind eye and sees its absence).

Another image of negation of a given thing (a negation that first of all negates the thing) is an image of a cross or of a slash (or of a sign in general) *erasing* such thing: not A is the cancellation (or the elimination, etc.) of A.

Again, thirdly, a negation of a certain thing could be visualized as another thing that is *external* with respect to that first thing: not A is external with respect to A (in this third case one can see why the idea of negation is sometimes called ‘logical complement’.)

The ideas of negation as i) absence of something and ii) erasing of something could be captured by the containment model. The idea of negation (‘not A’) as iii) external thing (with respect to A) could be captured by the entailment model.

Here are the three images of negation that we have just mentioned: (Image 30)

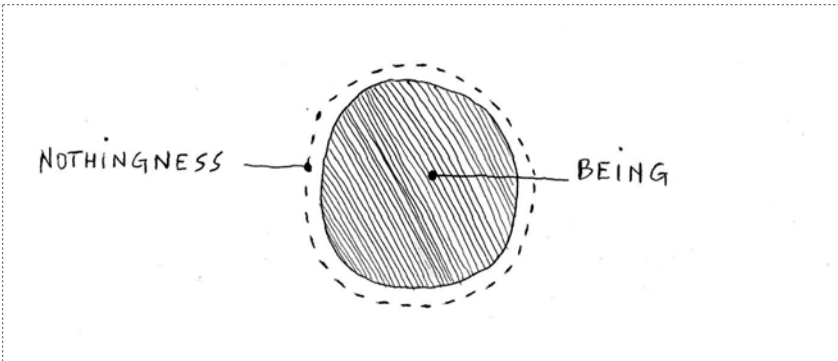


31

About the idea of negation, in particular about the idea of absolute or independent negation, one should observe this: that if one takes i) something (A) *and* ii) a negation (N) to refer to *absolute* things, one has i) being and ii) nothingness (here an absolute property would also become an absolute thing or subject).

One sees here that being and nothingness *could exist together at the same time and in the same respect*. This is so because the image of nothingness, as something independent, does not corrode the image of being (similarly: '0 + 1 = 1').

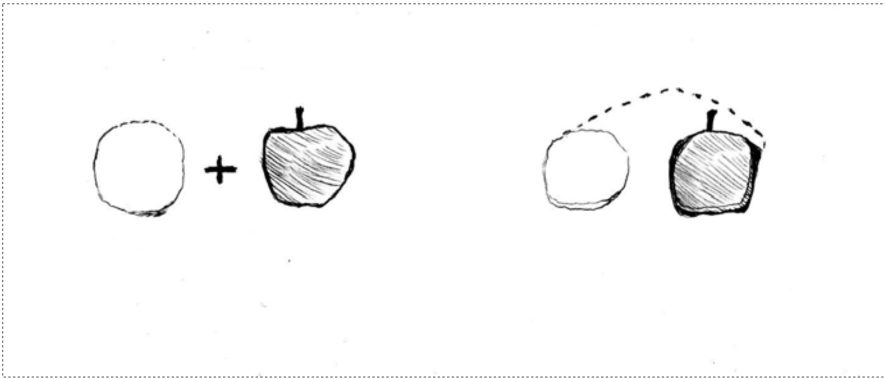
Here is a possible drawing of being and nothingness: (Image 31)



32

An image of conjunction between two (or more) things is the image of two (or more) things taken *together*: one could also see the idea of conjunction as, for example, two things that meet: for example as two segments (two streets, etc.) that meet. This latter image could also be used intuitively to explain the shape of the logical symbol '∧'.

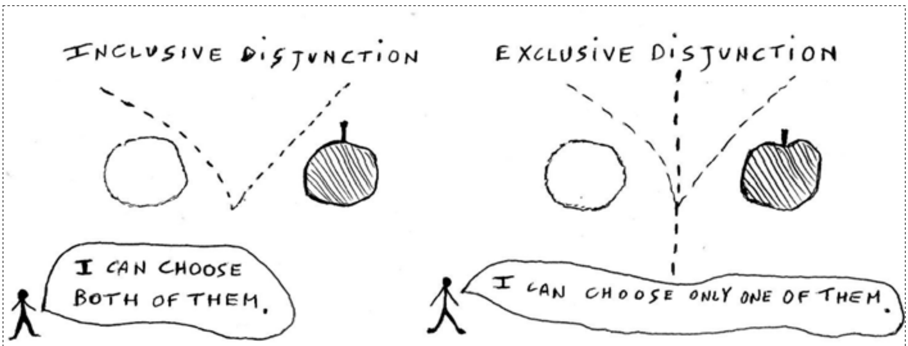
If, for example, one draws the proposition 'There are an orange and an apple' one has this: (Image 32)



33

One could visualize a logical disjunction by imaging two things that diverge, for example by imaging two segments that diverge, or two streets that bifurcate, etc. (this would intuitively explain the logical symbol ‘∨’, which however comes from the Latin word ‘vel’: ‘or’). The idea of *inclusive* disjunction refers to the case in which one could also take both the diverging segments, or both the diverging streets, etc. The idea of an *exclusive* disjunction refers to the case in which one has to make a real, hard choice – if one chooses, for example, the left segment, or the left street, etc., one cannot then also choose the right segment, or the right street, etc.

If one draws the proposition ‘There are an orange or an apple’ one has this: (Image 33)



(An intuitive symbol that has also been employed to represent a logical disjunction is this:

|

If one puts two objects-points on the left and right sides of such sign of disjunction one sees this:

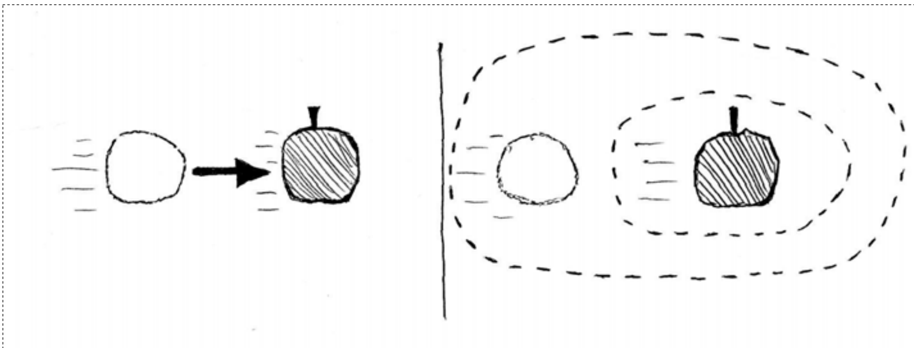
• | •)

34

A ‘material implication’ (or a simple conditional) is captured, for example, by the image of a ball causing the movement of another ball – this second ball is assumed, at the beginning, to be at rest. The idea behind the notion of material implication is the idea of something – sometimes called antecedent – causing something else: an effect, etc. – sometimes called consequent.

Another image that captures the idea behind the simple conditional is the image of containment. For example: one could visualize a material implication as a bigger circle that contains a smaller circle. When the smaller and bigger circles are both true, one sees that the bigger circle indeed contains, and thus implies, the smaller circle; when the bigger circle is true and the smaller circle false one sees that it cannot be the case that the bigger circle contains, and thus implies, the smaller circle; when the bigger circle is false one could imagine, about the smaller circle, whatever one likes in Latin, *ex falso quodlibet*.

Here is an image of an orange that hits and pushes an apple, and thus causes it to move. Here one says this: ‘If the orange moves, then the apple moves’. (Image 34)



35

The material implication has sometimes been expressed by means of the following symbol of entailment: ‘⊃’ (e.g. ‘A ⊃ B’, to be read as ‘A implies B’). If one sees the material implication as containment, one could however more precisely represent it by means of a ‘C’ symbol: ‘A C B’, to be read as ‘A contains B’. The relevant point is now this: one should clearly distinguish a form of inference as entailment (‘A entails or implies B’: ‘A ⊃ B’) from a form of inference as containment (‘A contains B’: ‘A C B’).

(About this point also see the notes in 52, 53 and 68.)

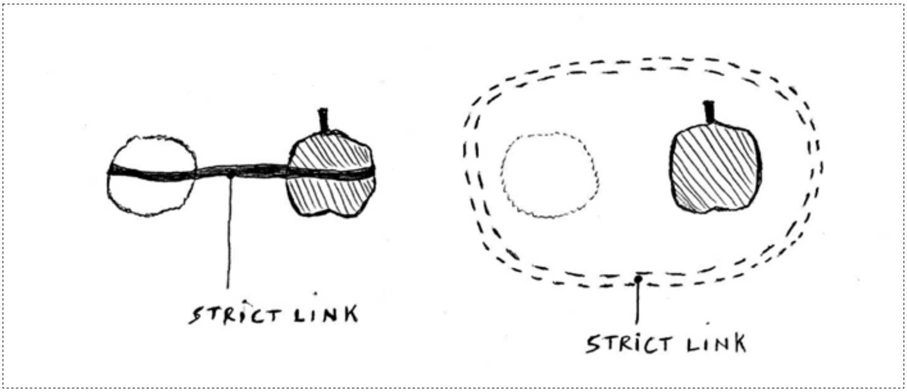
36

The idea behind the notion of (strict) logical implication, also called double conditional (biconditional), is the idea of two things, for example two objects, kept together by some kind of indissoluble link or tie. Since the link or tie between the two things is imagined to be indissoluble, when the first thing is true also the second thing is true; and when the first thing is false also the second thing is false - if the first thing were true and the second false or vice versa, one could not see that such things are kept together by an indissoluble link or tie.

Another appropriate image for the strict logical implication is that of two wholly overlapping or congruent circles.

The biconditional ‘p if and only if q’ is logically equivalent to the expression ‘p implies q and q implies p’.

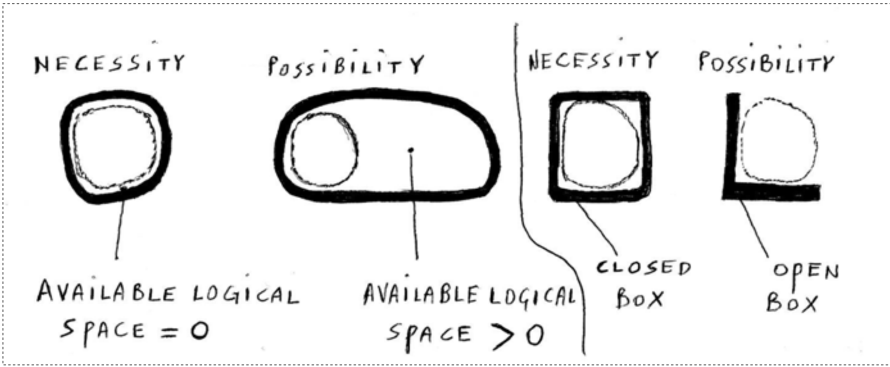
If, for example, one draws the proposition ‘There is an orange if and only if there is an apple’ one has this: (Image 36)



37

The idea of necessity in modal logic could be captured by showing that there is only one logical space (or one ‘possible world’, etc.) inside which something is the case. If one wants to draw the sentence ‘It is necessary that it is sunny’, one could draw a single circle that perfectly contains a point – the point, in this example, represents the sunny weather. Instead: the idea of possibility in modal logic could be captured by showing that there is more logical space than the space strictly containing what is the case (one usually searches this further logical space by using what is sometimes called ‘counterfactual imagination’). If one wants to draw the sentence ‘It is possible that it is sunny’, one can draw at least another circle close to the circle that strictly contains a point – i.e. that strictly contains the sunny weather. In fact: if it is possible that it is sunny then there must be a logical space that can contain the sunny weather, but also at least another logical space (that is, some more logical space) that can for instance contain a rainy weather, or a foggy weather, etc.

Another way of drawing ‘necessity’ and ‘possibility’ would be to show a close logical space (‘it is necessary that *p*’); and an open logical space (‘it is possible that *p*’). (Image 37)



(The standard signs used to represent necessity and possibility in modal logic are the following: necessity, '□'; possibility, '◊'. One might say that, sticking to the necessity sign, a more intuitive or impressionistic way of representing the ideas of necessity and possibility is this: necessity, '□'; possibility, '◊'.

38

Saul Kripke claims that rigid designators should be thought as proper names. Here I propose this: to think of rigid designators as *genuinely distinctive forms*, for example as *essential or identificational images* (fingerprints, etc.).

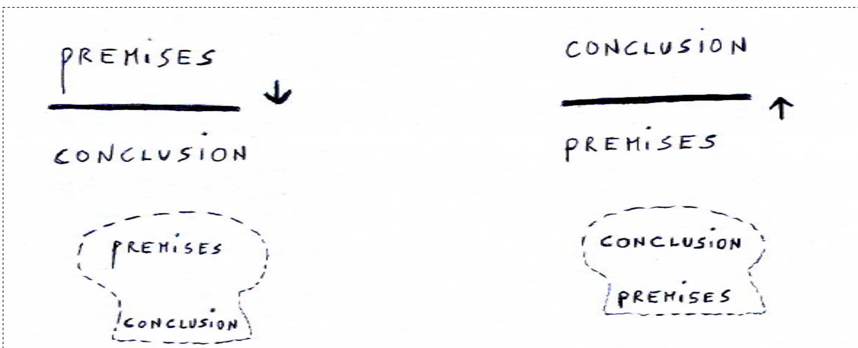
39

Each little space in the graph of a 'truth-table' could be seen as a 'possible world', that is, as a possible state or stage of the world.

40

A classic way to signal, in an argument or reasoning (e.g. in a syllogism), the distinction between premise or premises and conclusion is drawing a horizontal line between them. The line serves also to signal that the conclusion (the 'result') 'follows' from the premises.

An image that could adequately express the relationship between premises and conclusion is a picture according to which the premises contain or entail the conclusion. (Image 40)



41

When one uses different parentheses in a logical scheme (or formula, etc.) one wants usually to show, among the other things, that the truth of what is external rests on the truth of what is internal. For example, the truth of '{• [• (•)]}' contains the truth of '{•}'; and the truth of '{•}' contains the truth of '[•]'; and the truth of '[•]' contains the truth of '(•)'. With a metaphor: the truth about a whole onion contains the truth of the content of the first most external layer of the onion; and the truth of the content of the first most external layer of the onion contains the truth of the content of the second most external layer of the onion; etc. With another methapor: the truth about a whole matrioska contains the truth of the first most external matrioska; and the truth of the first most external matrioska contains the truth of the second most external matrioska; etc. (Image 41)

42

At the vertex of his logic Aristotle puts the principle of non contradiction (PNC, as discussed in particular in *Metaphysics* IV):

"*It is impossible for the same thing to belong and not to belong at the same time to the same thing and in the same respect.*" (*Metaph* IV, 3, 1005, b19-20.)

This is how modern symbolic logic writes the principle of non contradiction:

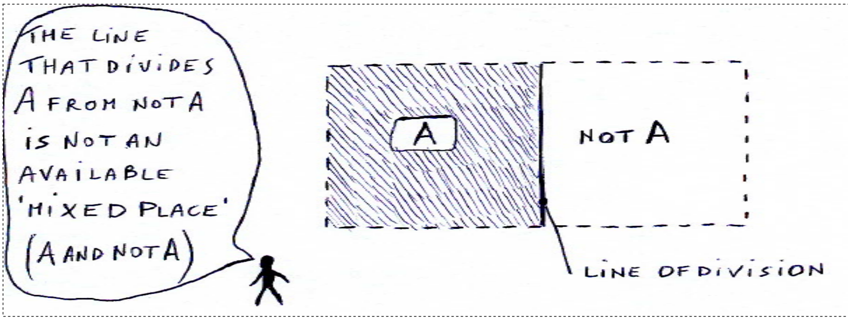
$\neg (A \wedge \neg A)$

(or: $\neg (A \;\&\; \neg A)$

(Etc.).

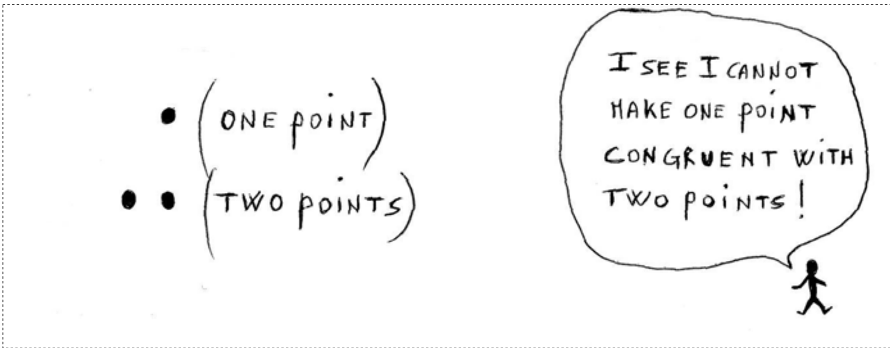
(Formulas to be read as: 'Not (A and not A)'.)

If one draws A and not A by means of some sort of Euler-Venn diagram, as in set theory, one aims to express this: firstly, that if something is in the 'A zone', then it cannot be in the 'not-A zone' (there cannot be cases that are (in) A and (in) not A at the same time (thus even the line or segment that divides the zone A from the zone not A in the diagram cannot be seen as a 'mixed place' (A and not A)). (Image 42)



43

If one would like to escape from the contradiction '1 = 2' one should be able to draw a (unit-) point '•' as completely congruent with two (unit-) points '••' (or viceversa). Here a person will in the end say: 'I see that I cannot make one point congruent with two points!' (Image 43)

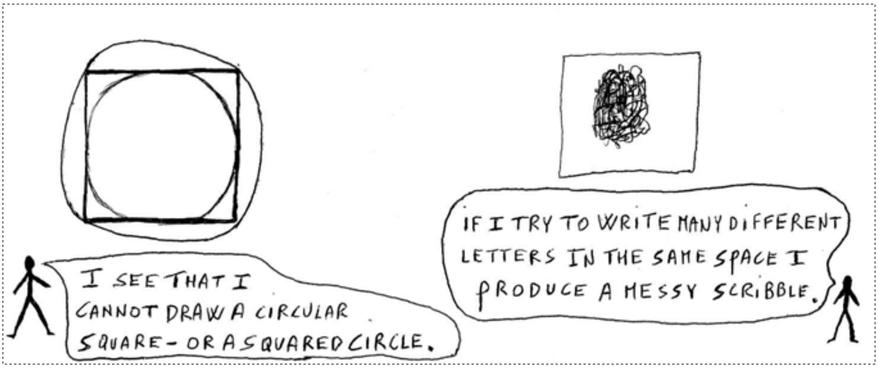


44

An attempt to arrive at an image of contradiction is, for example, a person's attempt to draw a squared circle – indeed one cannot imagine to put a circumference exactly inside a squared line or vice versa (indeed one cannot imagine to draw a circular square).

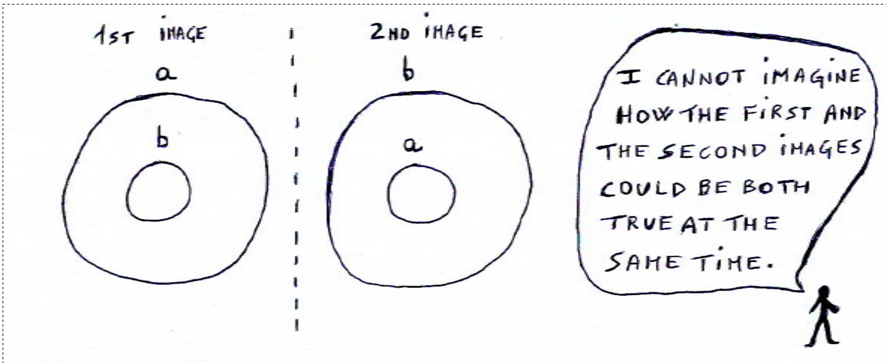
Here is another case concerning an attempt to get close to a contradiction: a person trying, for example, to write different alphabetical letters ('a', 'b', 'c', 'd', 'f', etc.) inside the very same space. The person sees, in this case, that the attempt to get close to a contradiction produces a messy scribble - the messy scribble is brought about by some sort of clash of incompatible or inconsistent images.

It has been said that a drunk (or drugged) person could for example have the experience of seeing a room as still and moving at the very same time. This experience should, in fact, be interpreted as a person's getting *close* to the idea of contradiction - a contradiction being, in fact, an experiential impossibility. (Image 44)



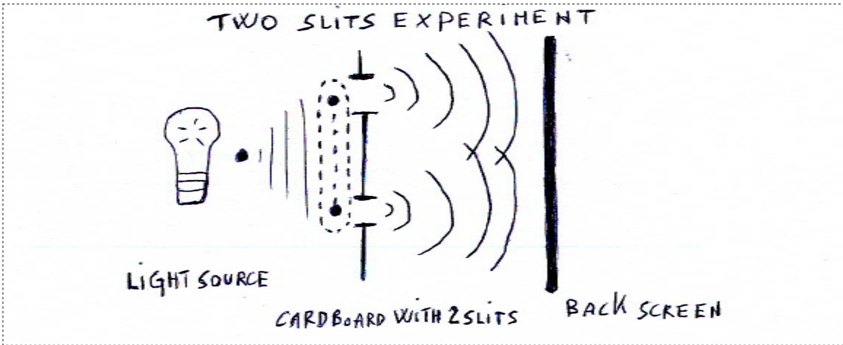
45

Perhaps the simplest way to display a contradiction is to show the impossibility of putting together the image of an object *a* that *contains* an object *b* with the image of an object *a* that *is contained by* an object *b*. (Image 45)



46

About the so called ‘quantum logic’: one could observe that the quantum phenomenon in physics (double-slit experiment, etc.) does not force one to abandon the principle of non contradiction. One could assume here that when a particle is said to be in two places at one time, it is in fact in a ‘unitary’ wider place: it is in the ‘single’ place corresponding to the absolute value of the sum of the two areas that are believed to contain it (so, for example: ‘The particle moves to the left (momentum) and is in the interval [0, 1] or in the interval [–1, 1]’ becomes ‘The particle moves to the left (momentum) and is in the interval [–1, 1]’. This way of logically describing the quantum phenomenon is justified by the fact that a particle could be seen as a wave, or as an induced field, etc. (as, for example, in the ‘wave’ or ‘electro-magnetic field’ interpretations of the microscopic behavior of light). (Image 46.)



47

On sintax (syntactic structures) and semantics: a *syntactically* consistent or non contradictory formal system could be (or become) *semantically* inconsistent or contradictory – hence it could in some cases become undecidable.

(*Syntactically* comes from the word ‘syntax’, from the Greek ‘συνταξις’ (‘sintaxis’), which refers to the arrangement or ordering of certain signs; *semantically* comes from the Greek word ‘σημαντικος’ (‘semantikos’), and thus from the word ‘σημα’ (‘sema’), which refers to a significant sign or meaning.)

Now suppose to have a formal system like this:

‘a, c, d, e, f, h, i, l, m, n, o, r, s, t, y, *’

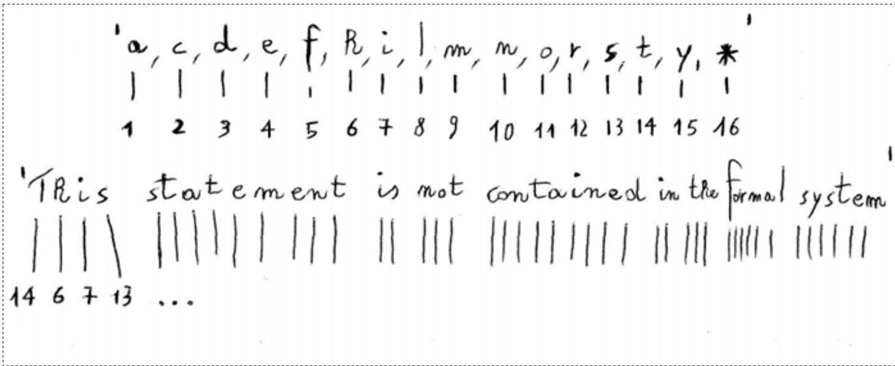
Suppose, then, to derive from the above signs - that is, from the above system of letters as sign-types - this:

‘This statement is not contained in the formal system’.

One can here see this: that syntactically (or structurally), the sign-types in the phrase ‘this statement is not contained in the formal system’ have been derived from the sign-types ‘a, c, d, e, f, h, i, l, m, n, o, r, s, t, y, *’: the formal system thus actually contains ‘this statement is not contained in the formal system’: the formal system clearly contains one sign-type more than ‘this statement is not contained in the formal system’: it contains all the types of letters of ‘this statement is not contained in the formal system’ *plus* the sign ‘*’. From a semantic point of view, however, ‘this statement is not contained in the formal system’ suggests something *opposite*: a meaning that denies the fact that the statement is contained in the formal system – a meaning that thus generates some kind of inconsistency or contradiction.

What does this case suggest? That syntactic consistency or syntactic non contradiction is not sufficient for semantic consistency or semantic non contradiction - meaning is something higher than syntax.

(Kurt Gödel’s incompleteness theorems should be seen to support similar conclusions.) (Image 47)



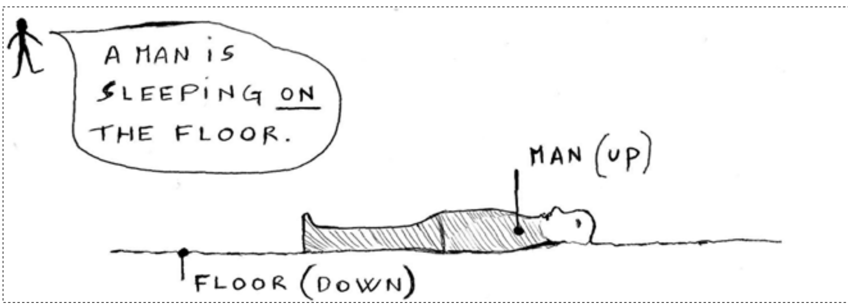
48

I take Gödel's incompleteness theorems to demonstrate this: that a given formal system cannot wholly *display* itself. In other terms: one point cannot represent more than one point - thus it cannot represent one point and the action needed for the point to see or be aware of itself as a point (this suggests that the mind's eye cannot be closed inside a system if it has to be possible for the mind's eye to watch the system).

49

One could notice that linguistic pr*e*positions such as 'of', 'in', 'on', 'through', etc. usually function, inside a language, to generate visual forms or visual structures. One can thus see that not only there is a 'pr*o*positional logic' (with 'o') but also a 'pr*e*positional logic' (with 'e'). By saying, for example, that 'a man is sleeping *on* the floor', one communicates, first of all, the following idea: that a sleeping man is placed in a higher – i.e. more northern - position than the floor.

Here one should then also take into account certain things such as spatial and temporal adverbs, etc. (temporal adverbs that would for example be relevant for a temporal logic, etc.). (Image 49)



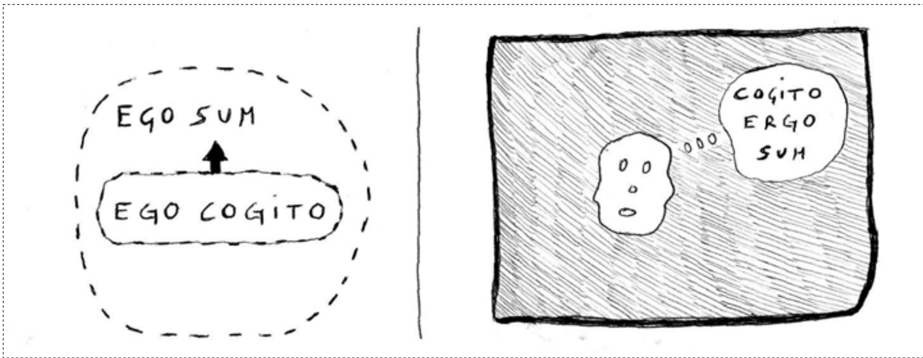
50

Some written symbols employed in the history of symbolic logic, and sometimes in the history of mathematics, are the following: '=': identity; '$\neq$': non identity; '$\neg$' (or '~', etc.): negation; '$\wedge$' (or '&', '.', etc.): conjunction; '$\vee$': disjunction (or '|'); '$\veebar$': exclusive disjunction; '$\rightarrow$' (or '$\supset$'): conditional (or implication); '$\leftrightarrow$'

(or '≡', etc.): double conditional (or material equivalence); '$\forall$': universal quantifier ('for every x'); '$\exists$': existential quantifier ('for some x'); '$\Box$': necessity; '$\Diamond$': possibility; '$\rightarrow$', 'therefore', usually pointing to a conclusion; '$\vdash$': 'yields' or 'proves', in proof theory; '$\cap$': intersection, in set theory; '+': addition; '÷': division; '∞': infinite; etc.
Of all these symbols, perhaps only a few could be held to be impressionistic (or at least somehow intuitive): '='; '$\neq$'; '$\Leftarrow$'; '$\rightarrow$'; '$\leftrightarrow$'; etc.

51

If one draws René Descartes' famous argument '*Cogito ergo sum*', one sees this: that the conclusion follows from the premise, for the premise visually entails the conclusion (this is so because the idea of '*res cogitans*' ('thinking thing') or '*cogitare*' ('to think') is wholly crossed by the transcendental idea of '*ens*' ('being') or '*esse*' ('to be')): (Image 51)



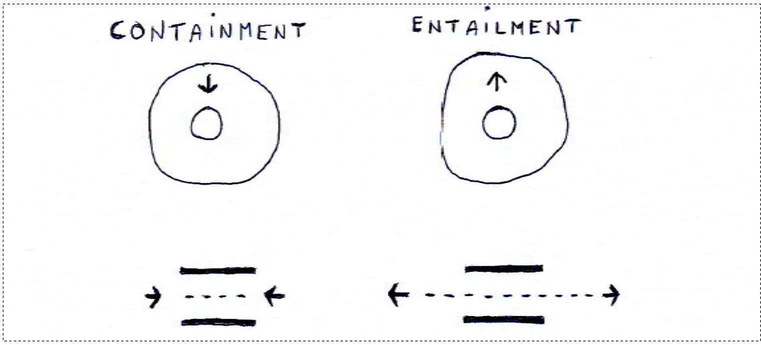
52

A premise i) could contain the conclusion, and thus give rise to a containment model; or a premise ii) could be crossed by a conclusion, and thus literally give rise to an entailment model.

Here are two examples.

First example: the reasoning 'If it is water, then it is H_20' refers to a figure of containment: the image in the premise 'it is water' contains the image in the conclusion 'it is H_20' (here I take this reasoning just to be expression of some kind of identity).

Second example: the reasoning 'If it is Moscow, then it is Russia' refers to a figure of (literal) entailment: the image in the premise 'it is Moscow' entails or implies the image in the conclusion 'it is Russia'. (Image 52)

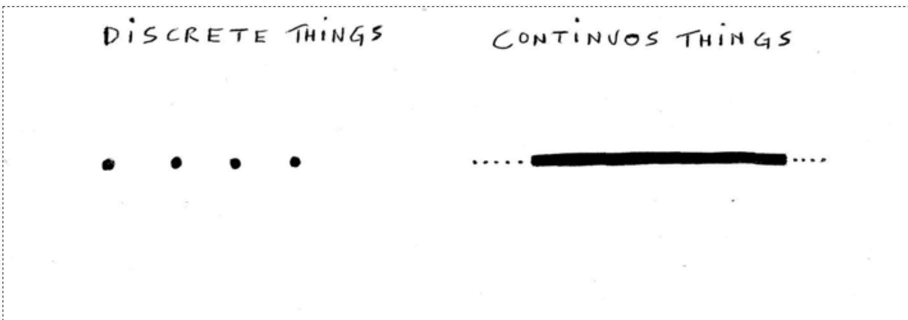


53

Theoretical logic is thus not only deductive, as in the containment model (where one deduces the conclusion which is contained in the premises); theoretical logic is also *extractive*, as in the entailment model (where one extrapolates the conclusion that crosses the premises)). Thus: one not only could 'derive' conclusions from premises, but also 'extract' conclusions from premises. Linguistic arguments – for example arguments based on a definition ('If he is a bachelor, then he is unmarried') - have usually the form of a containment. Transcendental arguments (as the 'cogito ergo sum' argument, etc.) have usually the form of an entailment.

54

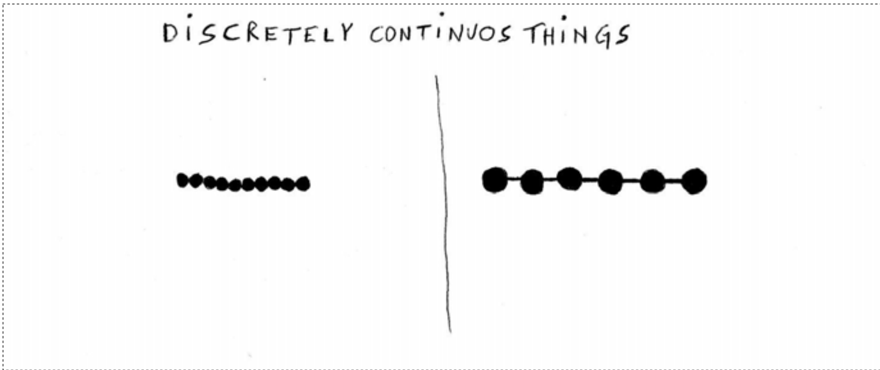
In philosophy of logic and mathematics, and indeed in logic and mathematics, it is important to keep in mind the image of something being discrete; and the image of something being continuous. Here are two possible drawings of such ideas: (Image 54)



55

How would it be something that is discretely continuous?

Here are two possible pictures of something that is discretely continuous: (Image 55)



56

A logic based on common images is the logic of a person who anchors her reasonings first of all on her reflections on her concrete, everyday visual experiences.

57

When one represents a logical case (a logical reasoning, etc.) by means of *dots* (*points*) – '•' – one more explicitly brings to light the form or structure underlying such case. In other words: if one uses dots to represent subjects/objects and properties one more explicitly sees those subjects/objects and properties.

58

A subject and a predicate - a subject that has a certain property, or attribute, or quality, etc. – could be drawn like this:

• •

Here one has however the problem that the dot representing the property is not symmetrically distributed with respect to the dot representing the subject. A better image would thus be this:

•

•

For example: if one would like to draw the proposition 'The dog is brown' one could draw this:

• ('Dog')

• ('Brownness')

If one would like even more explicitly to show that the property of brownness symmetrically applies to the dog, one could draw this:

• ('Brownness') • ('Dog') • ('Brownness')

For reasons of simplicity (and also again of symmetry) one could draw 'The dog is brown' (that is, 'The dog has the property of brownness') by means of a single point:

•

('Dog with brownness')

59

If one combines dots and basic logical symbols one has the following images (here one also takes the idea of a thing, that is of ‘something’, as basic – it means that this idea could contain the notions of subject, object and property):

Image of one thing:

•

Image of two things:

• •

Image of negation of a thing:

⌐ • (not •)

Image of conjunction of two things:

• ∧ • (• and •)

Image of disjunction of two things:

• ∨ • (• or •)

Image of conditional - as (necessary) relation that links two things:

• → • (if • then •)

Image of biconditional - as (necessary) stricter relation that links two things:

• ↔ • (if and only if • then •)

Image of identity (as isomorphism) (‘a = b’):

• = •

Another, stricter, image of identity (as numerical sameness) (‘a = a’):

= • =

Image of contradiction:

(• & ⌐ •) (• and not •)

Image of non contradiction:

⌐ (• & ⌐ •)

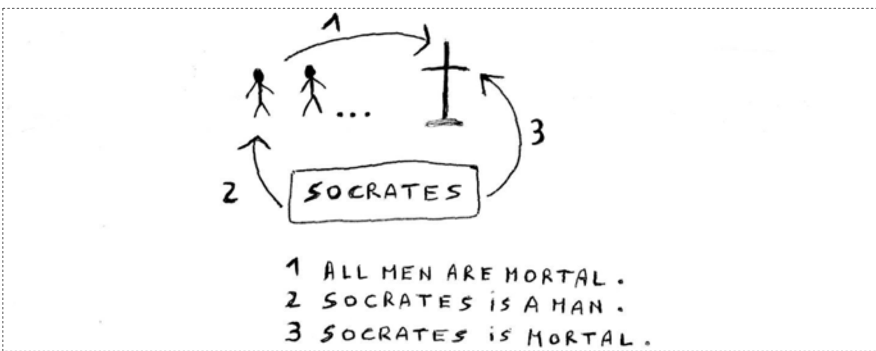
Image of equivalence between a thing and its double negation:

• = ⌐ ⌐ • (or: • ↔ ⌐ ⌐ •)

(…)

60

Let’s now observe a possible image corresponding to a famous syllogism: ‘All men are mortal. Socrates is a man. Thus Socrates is mortal’ (‘All men are mortal’ is the first premise. ‘Socrates is a man’ is the second premise. ‘Thus Socrates is mortal’ is the conclusion.): (Image 60)



61

About quantification in logic: in his theory of syllogism, Aristotle introduces four basic ideas concerning quantification in logic. These main four ideas are associated with what are known as 'A, E, I, O' subject-predicate propositions. They are:

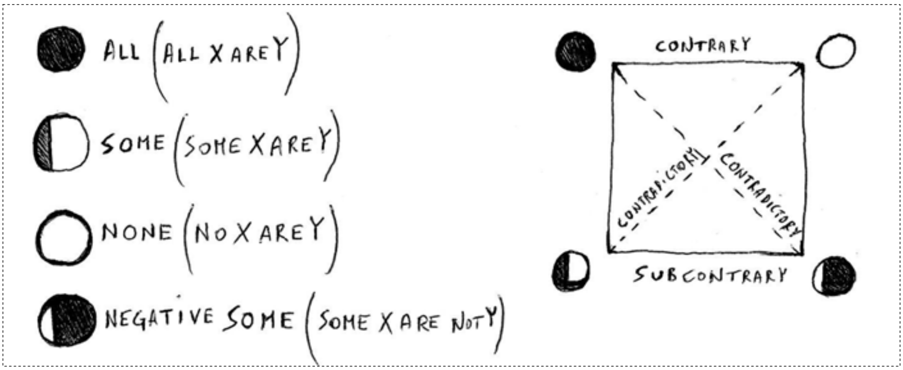
'A': '*All*'. As in: 'All X are Y' (universal affirmative proposition).

'E': '*None*'. As in: 'No X are Y' (universal negative proposition).

'I': '*Some*'. As in: 'Some X are Y' (particular affirmative proposition).

'O': '*Negative Some*'. As in: 'Some X are not Y' (particular negative proposition).

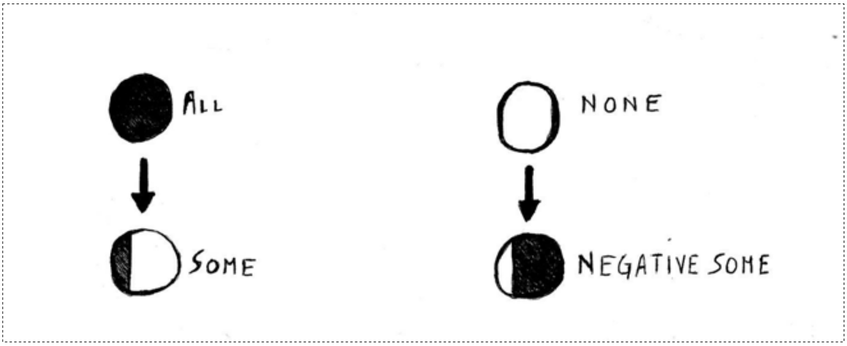
One could draw Aristotle's four ideas or 'forms' of quantification as follows: (Image 61)



62

Quantificational logic: one could derive the idea of 'some' from the idea of 'all' - the idea of 'all' contains the idea of 'some'; similarly, one could derive the idea of 'some do not' from the idea of 'none' - the idea of 'none' contains the idea of 'some do not' (in Aristotle's terms, the idea of 'some' is *subaltern* to the idea of 'all'; and the idea of 'some do not' is *subaltern* to the idea of 'none').

About the main rules of inference in quantificational logic: one can eliminate a universal quantifier and keep a concrete constant because the universal quantifier at least *contains* a concrete constant. One can introduce a universal quantifier because the sum of all the concrete constants *entails* the universal quantifier. Similarly, one can eliminate an existential quantifier and keep a concrete constant because the existential quantifier at least *contains* a concrete constant. One can introduce an existential quantifier because one concrete constant *entails* the existential quantifier. (Image 62.)



63

Let's now come back to the so called propositional logic. If one conceives the truth-tables in a more visual way one moves, for example, from the negation table (1) to the negation table (3):

(1)

P	Not p
T	F
F	T

(2)

•	Not •
T	F
F	T

(3)

T	F
•	Not •
Not •	•

64

Here are all the main truth-tables (for negation, conjunction, negative conjunction, disjunction, negative disjunction, conditional and biconditional) if one conceives them in a visual or more visual way:

T	**F**
•	Not •
Not •	•
• And •	Not • And •; • And Not •; Not • And Not •
Not • And Not •	• And •; Not • And •; • And Not •
• Or •	Not • Or Not •
Not • Or Not •	• Or •
If • then •	If • then Not •
Iff • then •	Iff Not • then •; Iff • then Not •

If one substitutes the word ‘not’ with the symbol ‘⌐’ in the above table one has this:

T	**F**
•	⌐ •
⌐ •	•
• And •	⌐ • And •; • And ⌐ •; ⌐ • And ⌐ •
⌐ • And ⌐ •	• And •; ⌐ • And •; • And ⌐ •
• Or •	⌐ • Or ⌐ •
⌐ • Or ⌐ •	• Or •
If • then •	If • then ⌐ •
Iff • then •	Iff ⌐ • then •; Iff • then ⌐ •

65

Now an example: is the proposition ‘Andrea is a boy and Hilary is a girl’ true? Here one has to keep in mind the image corresponding to the (positive) true conjunction:

‘• And •’

If it is true that ‘Andrea is a boy’ one has, to begin, this: •

If it is true that ‘Hilary is a girl’ one has, to begin, this: •

Here one has ‘• And •’, and thus the proposition ‘Andrea is a boy and Hilary is a girl’ is true.

Let’s now consider this other example: is the proposition ‘Andrea is not a boy and Hilary is not a girl’ true?

Here one has to keep in mind the image corresponding to the (negative) true conjunction:

‘⌐ • And ⌐ •’

If it is true that ‘Andrea is not a boy’ one has, to begin, this: ⌐ •

If it is true that ‘Hilary is not a girl’ one has, to begin, this: ⌐ •

Here one actually has ‘⌐ • And ⌐ •’, and thus the proposition ‘Andrea is not a boy and Hilary is not a girl’ is also true.

*

Let's now notice this: if one employs the traditional, non visual, truth-table for the conjunction (as in 63 (1)) for evaluating the propositions 'Andrea is a boy and Hilary is a girl' and 'Andrea is not a boy and Hilary is not a girl', one tends immediately to write this (since they are both true, in different cases):

'T And T' ('Andrea is a boy and Hilary is a girl')

'T And T' ('Andrea is not a boy and Hilary is not a girl')

So here one has, for example, that 'Andrea is a boy' and 'Andrea is not a boy' are *immediately* represented in the very same way: one has here that 'p' is immediately represented just as T (true) and 'not p' is also immediately represented just as T (true). This is somehow irrespectful even of the traditional truth-table for the negation, for the traditional truth-table for the negation at least *formally* displays a 'p' (T) *distinct* from a 'not p' (T). The problem with the traditional truth-table for the negation (as in figure 63 (1)) is thus this: that it induces one to conflate different cases together and makes one think that 'not p' is always automatically dependent on 'p' – it somehow induces one to think of 'not p' *just functionally or algorithmically*.

(Of course: if one aims visually to demonstrate that a given logical situation contains (or does not contain) a contradiction one should in the end come to work with a drawing where the truth is always represented as truth presence ('•') and the falsity as truth absence ('⌐ •')).

66

One main idea suggested in the positive or constructive part of these notes is the following: it seems to be important to draw a variable or its instantiation (here seen as an empirical 'something' or *quidditas*) as a point/dot:

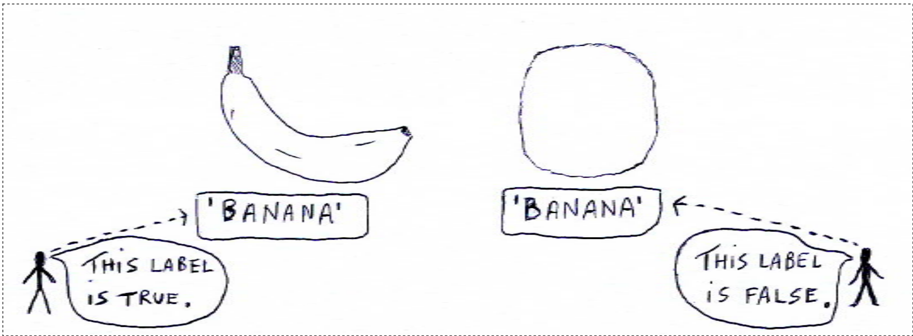
•

By representing things in terms of points/dots one uncovers their minimal form.

67

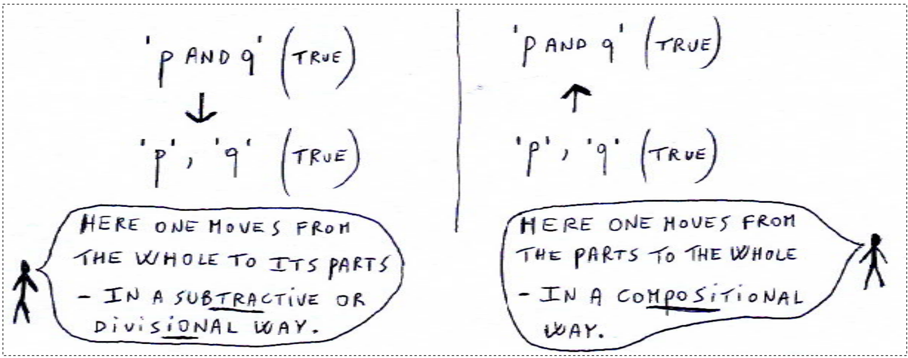
About the idea of logical atomism – in Russell, etc. – one can observe this: that an '*atomic' proposition* (or 'basic' singular proposition) could even be conceived as a mere subject proposition - not as a subject-predicate proposition, as the classic logical atomist would maintain. The case of a mere subject proposition is of course an extreme or limit case, though it seems to be important for logic to recognize such limit or basic case.

An example of a subject proposition, a one argument meaningful proposition, is the single name that one finds as entry in a dictionary; or it is the single name that one sometimes finds displayed as label of a certain thing: a good, a person, etc. (the singular proposition accompanies in this latter case an image (the proposition plays in this case a role similar to that one of a contracted demonstrative sentence such as 'This (is this)', or 'I (am I)', etc.)). For instance, the word 'banana' displayed as label on a banana is normally seen or interpreted to held a true value (and the word 'banana' displayed as label on an orange is instead normally seen or interpreted to hold a false value). (Image 67).



68

If one sees the truth tables from a genuinely *deductive*, that is, subtractive or divisional, perspective one sees, for example, that ‘p’ is true and ‘q’ is true because ‘p and q’ is true - here the truth of ‘p and q’ contains the truth of ‘p’ and the truth of ‘q’ (likewise: it is true that I have one parent and another parent because it is true that I have two parents). On the other hand, one could see the truth tables in a *compositional* way: one could see, for example, that ‘p and q’ is true because one has defined this to be true when ‘p’ is true and ‘q’ is true - here the truth of ‘p’ and the truth of ‘q’ do not contain but literally entail the truth of ‘p and q’. (Image 68)



69

According to Leibniz, logic requires a ‘*lingua generalis*’. Here I have instead suggested this: that logic requires a ‘*pictura generalis*’.

According to Boole, logic requires an algebraic system. Here I have instead suggested this: that logic requires a (human) geometrical view.

According to Frege, logic requires a ‘concept-script’ or a ‘concept-writing’ (a ‘*Begriffsschrift*’, in the original German). Here I have instead suggested this: that logic requires an iconography, that is, in its minimal form, a ‘pointography’ (or ‘dottography’).

70

What is truth? What is falsity? The ideas of truth and falsity play a crucial role in philosophy and logic. Here I will simply say this: truth is seeing (or ‘seeing’) that something is the case (i.e. that something is a fact); and falsity is seeing (or ‘seeing’) that something is not the case (i.e. that something is not a fact).

So for example: i) it is true that 'it rains' presupposes that one can actually see that it rains; ii) it is true that 'it does not rain' presupposes that one can actually see that it does not rain; iii) it is false that 'it rains' presupposes that one can actually see that it does not rain; iv) it is false that 'it does not rain' presupposes that one can actually see that it rains.

Let's now consider three ideas of truth by three logicians-philosophers: Aristotle, Tarski and Kripke.

71

In his *Metaphysics* (1011b25), Aristotle famously claims that "to say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, and of what is not that it is not, is true". Now the difficulty with this idea of truth is that it does not put into focus *how* we know what is and what is not. A more substantial idea of truth and falsity would thus make explicit this: that truth is seeing that what is is and falsity is seeing that what is not is not (in other works – e.g. *Categories* – Aristotle seems to come closer to such idea of truth and falsity).

72

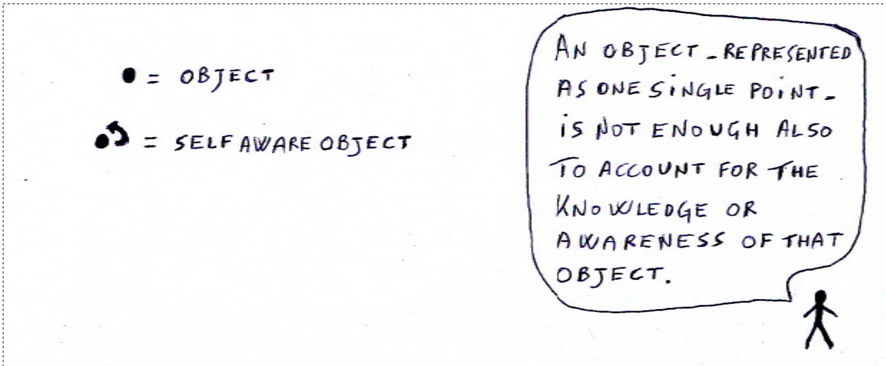
For Tarski, truth is captured by the following formula: φ (s) if and only if ψ. Such formula should be understood in this way: all the time in which one has a name s (e.g. 'snow is white') for a sentence S in a Language L, s is true (φ) if and only if one has a copy ψ of S (e.g. snow is white) in a Metalanguage M. In other words: s is true if and only if ψ is the object that 'satisfies' s. Given what we have just said, Tarski's account of truth is sometimes described as 'relation of satisfaction', a relation at the centre of a 'semantic conception of truth'.

A problem with such interpretation of truth is that Tarski in not capturing here the idea of truth but instead a formal relation of satisfaction. Tarski's reading of truth is too abstract - it is, at most, an indirect conception of truth. The sentence 'snow is white' is true if and only if snow is white just implies an act of disquotation. But one does not know if a sentence is true just by removing from it the signs of quotation (i.e. " "). In order for one to be able to know whether a sentence is true one has to watch the world referred to by such sentence.

73

Kripke's account of truth has some points of similarity and difference with respect to Tarski's, though in the end Kripke's idea of truth signals that he would like to take the opposite direction than that suggested by Tarski. A point of similarity between Kripke and Tarski is for example that both their conceptions of truth are based on the idea of relation of satisfaction (or saturation, etc.). Differences: Kripke's account of truth is linguistic in spirit, at least much more linguistic than Tarski's. Why? Because Kripke's main intention is that of trying to bring the true predicate, to be applied to a sentence, inside the Language containing the sentence - it is this that signals that Kripke's view of logic moves in the opposite direction than Tarski's (who, as we have said, opens his view of what is true by exploiting the idea of an explicit upper order, a Metalanguage).

Kripke's linguistic move is of course coherent with his general ambition to reinvigorate a classic philosophy of language. Such move, however, makes truth, that is the source of truth, once again wholly implicit. Indeed: by bringing truth inside the Object Language, Kripke makes the source of truth – the knowing subject – completely hidden inside the object (from this it follows that Kripke's idea of the 'fixed point' is unnecessarily artificial). Evidence of Kripke's doubts about the plausibility of his linguistic proposal can be found in a claim by Kripke himself. Indeed, towards the end of his *Outline of a Theory of Truth*, he writes: "The ghost of the Tarski's hierarchy is still with us". (Image 73)



74

Significant tests for the comprehension of logic are the so called logical paradoxes.

One main family among the logical paradoxes is that of the semantic paradoxes. Perhaps the most discussed semantic paradox is the Liar's Paradox, the clearest version of which is the proposition or sentence 'This statement is false'.

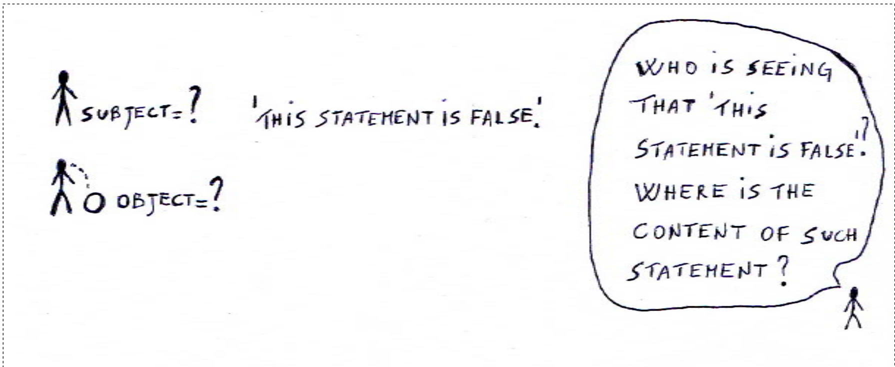
Now what are the difficulties with such proposition? The statement seems to imply a paradox because if one says that it is false, then it is true.

But let us now observe better this proposition.

The first point that one should notice here is that one does not know *who* – which conscious subject – is concerned with the proposition 'This statement is false'. If one takes the proposition at its face value it seems that it is the statement itself that is saying of itself that it is false (let's of course observe here that we do not have 'This statement' is false). A proposition – a statement –, however, cannot decide about its truth or falsity! A proposition does not have experiences, nor think, etc.

The second point that one should notice about this case is this: that one cannot see the content of 'This statement is false'. Indeed: if one takes |This statement is false| to be the fact corresponding to 'This statement is false' one should claim that 'This statement is false' is true. But now: if one takes |This statement is true| to be the fact corresponding to 'This statement is true' one should claim that 'This statement is true' is also true. Here one would thus have this impossible fact: |This statement is both false and true|. (Notice here that the problem is not with the ostensive claim introduced by the demonstrative 'this'. If one should write: 'This word has four letters' one could interpret this statement as true: This 'word' has indeed four letters, and one could indeed count them, etc.)

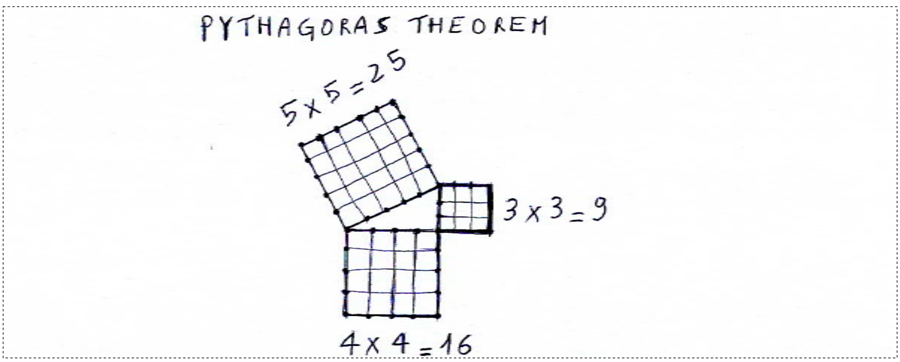
'This statement is false' is thus not false nor true: it is *pre-epistemic*. (Image 74)



75

At the heart of logic there is the idea of proof. A substantial proof is a demonstration: a demonstration that allows one to see that truth is preserved while passing from one or more premises (or assumptions) to a conclusion. At the basis of a proof there is a comparison of ideas, and more generally a reflection on ideas (here conceived as visible experiences). In *An Essay Concerning Human Understanding*, Locke claims: "Those intervening ideas which serve to show the agreement of any two others are called 'proofs'; and where agreement or disagreement is by this means plainly and clearly perceived, it is called demonstration".

What one usually does while attempting to prove something is thus simplifying it – usually one simplifies the two images on both sides of the identity sign, so that to create the possibility of displaying – demonstrating – that they are indeed congruent (or indeed incongruent). In some cases, one might have to elaborate the two images on both sides of the identity sign so to make their congruence (or incongruence) wholly *explicit*. Perhaps the most paradigmatic example of a proof that makes explicit the congruence between two expressions is the drawing that demonstrates the Pythagorean Theorem: '$a^2 + b^2 = c^2$'. (Image 75)



76

At the beginning of logic one should just put one axiom: the mind eye's axiom. It is a person's ability of seeing or intuiting different forms or structures inside the logical space – and then making them publicly visible – that constitutes the basis of logic. One could in general describe such ability as a person's capacity of logical vision.