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TESTING SIGNIFICANCE OF RANDOM EFFECTS FOR THE GAMMA DEGRADATION MODEL

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Gamma degradation models with fixed or random effects are widely used for reliability analysis. In this paper, the problem of testing significance of random effects for the gamma degradation model is considered. We propose two statistical tests which enable to reveal the existence of random effects in degradation data corresponding to the gamma degradation model. The first test is the well known likelihood ratio test and the second one is based on the variance estimate of the random parameter of the "random-effect" gamma degradation model. These tests have been compared in terms of power with Monte-Carlo simulation method. Moreover, the example of GaAs lasers degradation analysis has been considered.
Keywords: gamma degradation model; fixed-effect model; model with random effects; reliability; GaAs lasers.

Statistical degradation models are used for the analysis of lifetime data of tested items in the cases when along with the failure time data there is the detailed information about the change of degradation index [1, 2]. Parametric models, which are distinguished by the distribution of increments of degradation index and the existence of random effects, are widely used in practice. In [3–5] and [6, 7], the authors consider the gamma degradation model with random effects, where the scale parameter is a random variable from the gamma distribution. Thus, considering the "random-effect" degradation model, we need to take into account the distribution of the random parameter and hence, the number of unknown parameters of the "random-effect" model is larger than the number of unknown parameters of the "fixed-effect" model. As a result, the accuracy of parameter estimation for the "random-effect" model may decrease. On the other hand, if the unit-to-unit variability is rather large, then the "fixed-effect" model is not appropriate, and in this case, the use of the "random-effect" model could provide more accurate estimates. By this reason, the use of the degradation model with random effects is not advisable when the random effect is insignificant or not observed at all. So, it is necessary to have the statistical test which can reveal the random effect influence and help with the choice between fixed- and random-effect degradation models.

In [8], the Hausman test is proposed for distinction between the "fixed-" and "random-effect" models. However, this statistical test is applied only for linear regression models where the estimates are calculated by the least square method that does not allow using the Hausman test for degradation models. Other criteria for comparison of statistical models are AIC and BIC information tests [9]. These tests are based on values of the maximum likelihood function and apply the penalty for test statistics value taking into account the number of estimated parameters. Such information criteria enable to compare statistical models, but they are not used for hypothesis testing. Hereby, it is necessary to develop a criterion which can test the hypothesis of absence of random effects in degradation data. So, the goal of this research is to develop a statistical test, which enables to reveal the existence of random effects in degradation data corresponding to the gamma degradation model.

In this paper, we propose two tests for the hypothesis of absence of random effects for the gamma degradation model: the likelihood ratio test and the test based on the variance estimate of the random parameter. Moreover, we use Monte Carlo simulations to investigate the power for the constructed tests for different pairs of competing hypotheses. Then, we illustrate testing significance of random effects for the gamma degradation model using the example of GaAs lasers data analysis, which is often considered in publications, devoted to the investigation of degradation models [4, 6, 9, 10]. In [4], the “random-effect” gamma degradation model was fitted. In [6], these data have been analyzed using gamma and Wiener degradation models. Inverse Gaussian degradation model is described in [9] as another variant of the degradation model for the lasers data.

1. Gamma degradation models

Stochastic process $Z(t)$ characterizing degradation process is referred to as the gamma degradation process, if

– $Z(0)=0$;
– $Z(t)$ is a stochastic process with independent increments;
– increments $\Delta Z(t)=Z(t+\Delta t)-Z(t)$ have the gamma distribution with probability density function:

$$f_{Gamma}\left(x;\sigma,\Delta\nu\left(t\right)\right)=\frac{x^{\Delta\nu(t)-1}}{\sigma^{\Delta\nu(t)}\Gamma\left(\Delta\nu\left(t\right)\right)}e^{-\frac{x}{\sigma}},$$

where $\Delta\nu\left(t\right)=\nu\left(t+\Delta t\right)-\nu\left(t\right)$ is the shape parameter and σ is the scale parameter, $\nu\left(t\right)$ is a positive increasing function [7].

If random variables ξ_1 and ξ_2 follow the gamma distribution with scale parameter σ and shape parameters ν_1 and ν_2, correspondingly, then $\xi_1+\xi_2$ follows the gamma distribution with scale parameter σ and shape parameter $\nu_1+\nu_2$. This property explains the fact of using the gamma distribution as a distribution of increments.

Let the mathematical expectation of degradation process $Z(t)$ is

$$M\left(Z\left(t\right)\right)=m\left(t\right),$$

where $m\left(t\right)=m\left(t;\gamma\right)$, $\gamma=\left(\gamma_1,...,\gamma_s\right)^T$ is a trend function of the degradation index. Then, the shape parameter is equal to $\Delta\nu\left(t\right)=\dfrac{\Delta m\left(t\right)}{\sigma}$.

In this paper, we consider two types of trend functions:

– linear function $m\left(t\right)=\gamma_1 t$, $\gamma_1>0$;
– power function $m\left(t\right)=\gamma_1 t^{\gamma_2}$, $\gamma_1>0$, $\gamma_2>0$.

Taking into account the given assumptions, the stochastic process $Z(t)$ at time moment $t=t_k$ has the gamma distribution with the shape parameter equal to $\nu(t_k)=\dfrac{m(t_k)}{\sigma}$.

The time to failure is the random variable

$$T=\sup\{t:Z(t)<z_0\},$$

where z_0 is the critical value of the degradation path. Then, the reliability function for the gamma degradation model is given by:

$$S(t)=P\{T>t\}=P\{Z(t)<z_0\}=F_{Gamma}\left(z_0;\sigma,\nu(t)\right). \quad (1)$$

As was noted in the introduction, if the unit-to-unit variability is rather large, then it is necessary to take into account the heterogeneity in degradation paths. In [3], the “random-effect” gamma degradation model is specified by considering parameter σ as a random effect. To obtain mathematically tractable distributions, it is assumed that the random parameter $\eta=\sigma^{-1}$ has the gamma distribution with the density function $f_{Gamma}\left(x;\delta^{-1},\theta\right)$, where θ is the shape parameter and δ^{-1} is the scale parameter. Here η has mathematical expectation $M\eta=\theta/\delta$ and variance $D\eta=\theta/\delta^2$, and σ has finite mathematical expectation $M\sigma=\delta/\left(\theta-1\right)$ for $\theta>1$ and finite variance

$$D\sigma = \frac{\delta^2}{(\theta-1)^2(\theta-2)}$$

for $\theta > 2$ [3]. Then the marginal density function for $Z(t)$ in the case of gamma degradation model with random effects is equal to:

$$f_{Z(t)}\left(x;\delta,\theta,\nu(t)\right) = \int_0^\infty f_{Gamma}\left(x;\omega^{-1},\nu(t)\right) f_{Gamma}\left(\omega;\delta^{-1},\theta\right) d\omega = \frac{x^{\nu(t)-1}\delta^\theta}{(x+\delta)^{\nu(t)+\theta}} \mathrm{B}^{-1}\left(\nu(t);\theta\right),$$

where $B(\cdot,\cdot)$ is the Euler beta function. The shape parameter of the gamma degradation model with random effects is $\nu(t) = \frac{(\theta-1)\cdot m(t)}{\delta}$. It can be noted that $\frac{\theta}{\delta\nu(t)} \cdot Z(t)$ has an F-distribution with parameters $2\nu(t)$ and 2θ. In this case, the reliability function can be written as

$$S(t) = P\{T > t\} = P\{Z(t) < z_0\} = \int_0^{z_0} f_{Z(t)}\left(x;\delta,\theta,\nu(t)\right) dx = F\left(\frac{\theta \cdot z_0}{\delta \cdot \nu(t)};\ 2\nu(t);\ 2\theta\right). \quad (2)$$

Let the realization of stochastic process $Z(t)$ for the i-th item is denoted as

$$Z^i = \left\{(0, Z_0^i = 0), (t_1^i, Z_1^i), \ldots, (t_{k_i}^i, Z_{k_i}^i)\right\},\ i = \overline{1,n},$$

where k_i is the number of time moments, in which the degradation index was measured. Then, the sample of independent degradation index increments can be written as:

$$\mathbf{X}_n = \left\{X_j^i = Z_j^i - Z_{j-1}^i,\ i = \overline{1,n},\ j = \overline{1,k_i}\right\}.$$

Maximum likelihood estimates (MLEs) of parameters σ and γ of the “fixed-effect” gamma degradation model are calculated by maximization of the likelihood function:

$$L(\mathbf{X}_n) = \prod_{i=1}^{n}\prod_{j=1}^{k_i} \ln f_{Gamma}(X_j^i;\sigma,\nu_j^i), \quad (3)$$

where $\nu_j^i = \nu\left(t_j^i\right) - \nu\left(t_{j-1}^i\right)$, $i = \overline{1,n}$, $j = \overline{1,k_i}$ are the values of shape parameter.

If $Z^i(t)$, $i = \overline{1,n}$ are the gamma degradation processes with random effects, then the likelihood function can be written as a multiplication of the joint density functions of increments X_j^i on the common random effect:

$$L(\mathbf{X}_n) = \prod_{i=1}^{n} f\left(X_1^i, X_2^i, \ldots, X_{k_i}^i\right) = \prod_{i=1}^{n}\int_0^\infty \left[\prod_{j=1}^{k_i} f_{Gamma}\left(X_j^i;\omega^{-1},\Delta\nu(t_j)\right)\right] f_{Gamma}\left(\omega,\delta^{-1},\theta\right) d\omega =$$

$$= \prod_{i=1}^{n}\left[\frac{\delta^\theta}{\Gamma(\theta)} \cdot \frac{\Gamma\left(\nu(t_k)\right)}{\left(Z_k^i + \delta\right)^{\nu(t_k)+\theta}} \cdot \prod_{j=1}^{k_i} \frac{\left(X_j^i\right)^{\Delta\nu(t_j)-1}}{\Gamma\left(\Delta\nu(t_j)\right)}\right]. \quad 4)$$

2. Testing hypothesis of absence of random effects

Let us assume that observed degradation paths are the realizations of the gamma degradation process. If the unit-to-unit variability is rather large, then random effects in these data can be significant and the “fixed-effect” model is not appropriate. So, it is necessary to test the hypothesis of absence of random effects, which means that the parameter σ in the gamma degradation model is not random:

$$H_0:\ D\sigma = 0.$$

In fact, the acceptance of this hypothesis will imply that data correspond to the “fixed-effect” model.

The competing hypothesis H_1 corresponding to the “random-effect” model is written as:

$$H_1: D\sigma > 0.$$

Let consider two statistical tests for the null hypothesis: the likelihood ratio test and the test based on the variance estimate. The likelihood ratio test (LR test) is usually constructed for distinguishing between two competing statistical models. The LR test statistic value is calculated as follows:

$$\lambda_n = \ln \frac{L(\mathbf{X}_n \mid H_1)}{L(\mathbf{X}_n \mid H_0)}, \tag{5}$$

where $L(\mathbf{X}_n \mid H_0)$ is the maximum value of the likelihood function (3) in the case of "fixed-effect" model, $L(\mathbf{X}_n \mid H_1)$ is the maximum value of the likelihood function (4) in the case of "random-effect" model. The testing hypothesis H_0 is rejected for large values of λ_n. According to the Neyman-Pearson lemma, the LR test is the most powerful criterion, when testing a simple hypothesis. However, the hypothesis is composite, so this test cannot be the best one.

As an alternative approach, we consider the variance estimate of the random parameter (VERP):

$$d_n = \frac{\hat{\delta}_n^2}{\left(\hat{\theta}_n - 1\right)^2 \left(\hat{\theta}_n - 2\right)}, \tag{6}$$

where $\hat{\theta}_n$ and $\hat{\delta}_n$ are the maximum likelihood estimates of the shape and scale parameters of the "random-effect" model (2), correspondingly. In Table 1, there are the means and standard deviations of estimates d_n, obtained by $N = 10000$ simulated samples from the "fixed-effect" and "random-effect" models. The true values of parameters for the "random-effect" model are $\theta = 10$, $\delta = 1{,}5$, $\gamma_1 = 0{,}002$, and for the "fixed-effect" model are $\sigma = 14$, $\gamma_1 = 0{,}002$. The time moments for measuring degradation were chosen as follows: $t_j^i = t_{j-1}^i + 250$, where $t_0^i = 0$, $j = \overline{1, k_i}$, $i = \overline{1, n}$, $k_i = 16$.

Table 1

Means (M) and standard deviations (SD) of estimates d_n

True model	Descriptive statistic	n = 5	n = 10	n = 20	n = 30	n = 50
"Fixed-effect" model	M	$2{,}55 \cdot 10^{-6}$	$2{,}23 \cdot 10^{-6}$	$1{,}95 \cdot 10^{-6}$	$1{,}79 \cdot 10^{-6}$	$1{,}12 \cdot 10^{-6}$
	SD	$4{,}08 \cdot 10^{-11}$	$2{,}19 \cdot 10^{-11}$	$1{,}36 \cdot 10^{-11}$	$1{,}02 \cdot 10^{-11}$	$7{,}53 \cdot 10^{-12}$
"Random-effect" model	M	$4{,}01 \cdot 10^{-3}$	$3{,}63 \cdot 10^{-3}$	$3{,}61 \cdot 10^{-3}$	$3{,}56 \cdot 10^{-3}$	$3{,}54 \cdot 10^{-3}$
	SD	$1{,}08 \cdot 10^{-2}$	$4{,}28 \cdot 10^{-3}$	$6{,}88 \cdot 10^{-3}$	$5{,}49 \cdot 10^{-3}$	$2{,}86 \cdot 10^{-3}$

As can be seen from Table 1, the means of variance estimate d_n obtained for the "fixed-effect" model tend to 0 with the sample size growth in contrast to the means obtained for the "random-effect" model, which tend to the true value of $D\sigma = 3{,}49 \cdot 10^{-3}$. Thus, the variance estimate d_n of the random parameter can be used as a test statistic for testing the hypothesis of absence of random effect. Let us refer this test to as the VERP test. Similar to the LR test, the hypothesis H_0 is rejected for large values of d_n.

The theoretical statistics distributions for the proposed tests are not known as there are a number of factors influencing the form of the statistics distributions: the method of model parameters estimation, the type of trend function, the values and the number of time moments of measuring degradation, the sample size and others. So, to apply the LR and VERP tests we use the parametric bootstrap method according to the following algorithm:

1. Generate a sample of increments from the "fixed-effect" model with parameters $\hat{\sigma}_n$ and $\hat{\gamma}_n$ according to the given time moments t_j^i, $i = \overline{1, n}$, $j = \overline{1, k_i}$; here $\hat{\sigma}_n$ and $\hat{\gamma}_n$ are the MLEs obtained from the source data.
2. Determine the MLEs of parameters σ and γ of the "fixed-effect" model from the simulated sample of increments using the likelihood function (3).
3. Determine the MLEs of parameters δ, θ and γ of the "random-effect" model from the simulated sample of increments using the likelihood function (4).

4. Calculate the test statistics, namely λ_n and d_n.
5. Repeat points 1–4 N times to obtain the empirical distributions $G_N(s \mid H_0)$ for each proposed test.
6. Calculate the *p*-values $\alpha_n = 1 - G_N(S_n \mid H_0)$, where S_n is a value of test statistic (λ_n or d_n), calculated from the source sample.
7. If α_n is less than the significance level α, then hypothesis H_0 is rejected.

3. Empirical power study of the LR and VERP tests

The test power $1-\beta$ is the probability to reject the null hypothesis H_0 with the significance level α when the competing hypothesis H_1 is true:

$$1-\beta = 1 - G\left(S_a \mid H_1\right).$$

Actually, the more powerful test is, the higher its ability to distinguish close competing hypotheses. We have carried out the investigation of the LR and VERP test power for various pairs of competing hypotheses through Monte Carlo simulations.

The estimates of test power have been obtained for different sample sizes, sets of time moments t_j, $j = \overline{1,k}$ and magnitudes of the random effect. The number of simulations used $N = 10\,000$. The estimates of tests power were calculated with the nominal significance level $\alpha = 0,01$.

In Table 2, the powers of the proposed tests are presented for different sets of time moments for measuring degradation:

$$T_1:\ t_j = t_{j-1} + 400,\ \text{where } t_0^i = 0,\ j = \overline{1,10},$$

$$T_2:\ t_j = t_{j-1} + 250,\ \text{where } t_0^i = 0,\ j = \overline{1,16},$$

$$T_3:\ t_j = t_{j-1} + 125,\ \text{where } t_0^i = 0,\ j = \overline{1,32}.$$

Under hypothesis H_0, samples of increments were generated from the “fixed-effect” gamma degradation model with the scale parameter $\sigma = 14$; and in the case of true hypothesis H_1 samples were generated from the “random-effect” model with parameters $\delta = 1.5, \theta = 28$. The linear trend function with parameter $\gamma_1 = 0.002$ was taken.

Table 2

The power estimates of the VERP and LR tests for different sets of time moments *T*

Time frequency	n = 5	n = 10	n = 15	n = 20
VERP test				
T_1	0,69	0,93	0,99	1,0
T_2	0,70	0,94	0,99	1,0
T_3	0,71	0,94	0,99	1,0
LR test				
T_1	0,67	0,92	0,99	1,0
T_2	0,69	0,93	0,99	1,0
T_3	0,70	0,95	0,99	1,0

As can be seen from Table 2, the power of both tests increases with the growth of the number of items n and the frequency of measuring degradation.

The second experiment has been designed to show, how the power of proposed tests changes depending on the magnitude of the random effect under competing hypothesis H_1. For this research, we consider different values of the shape parameter: $\theta_1 = 42$, $\theta_2 = 35$, $\theta_3 = 28$ with the scale parameter $\delta = 1,5$, which correspond to different magnitudes of the random effect, as the variance $D\sigma$ decreases with the shape parameter growth. Time moments for measuring degradation index were taken corresponding to values of T_2 from the first experiment. In Figures 1-4, there are the examples of generated degradation paths according to the “random-effect” gamma degradation model with different values of shape parameter and the “fixed-effect” gamma degradation model. As can be seen from Figures 3 and 4, in the case of the “random-effect” model with $\theta = 42$ the unit-to unit variability looks very similar to the case of the “fixed-effect” degradation model, and it is difficult to distinguish these cases without a special statistical test.

In Table 3, the estimates of power of the proposed tests are presented for different values of shape θ of the random parameter σ and number of tested items n. The estimates of tests power were calculated with the nominal significance level $\alpha = 0{,}01$.

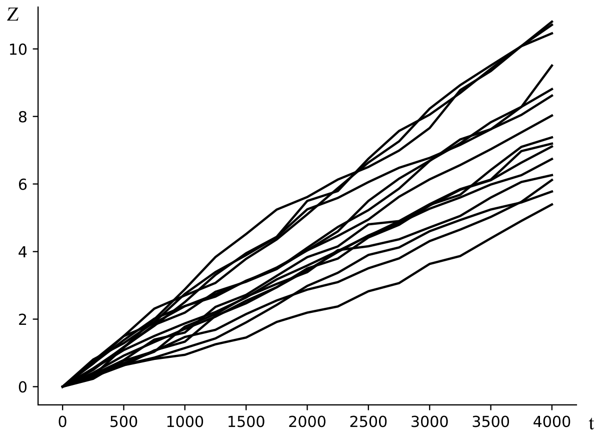

Fig. 1. The degradation path with the second experiment conditions and $\theta = 28$

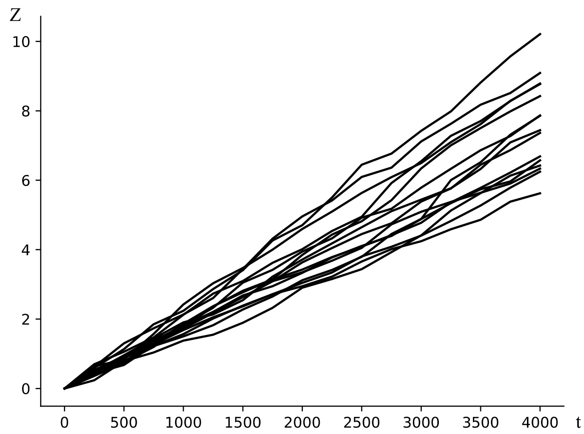

Fig. 2. The degradation path with the second experiment conditions and $\theta = 35$

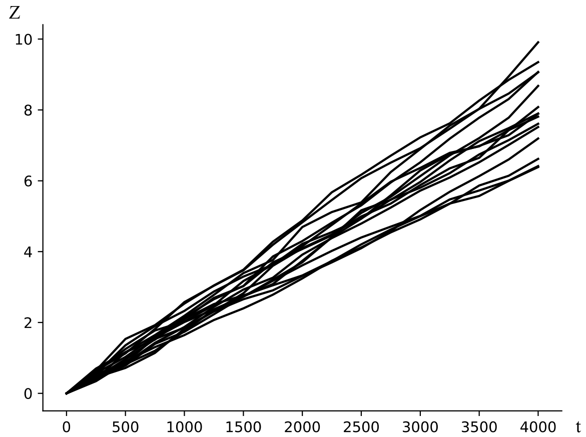

Fig. 3. The degradation path with the second experiment conditions and $\theta = 42$

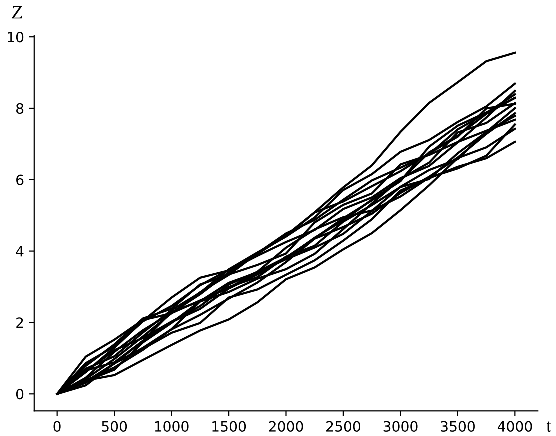

Fig. 4. The degradation path with the second experiment conditions for the “fixed-effect” model

Table 3

The power estimates of VERP and LR test for different values of shape θ of the random parameter σ and number of tested items n

Shape parameter	n = 5	n = 10	n = 15	n = 20
VERP test				
$\theta_1 = 42$	0,69	0,94	0,99	1,0
$\theta_2 = 35$	0,69	0,94	0,99	1,0
$\theta_3 = 28$	0,70	0,94	0,99	1,0
LR test				
$\theta_1 = 42$	0,67	0,93	0,98	0,99
$\theta_2 = 35$	0,68	0,93	0,98	0,99
$\theta_3 = 28$	0,69	0,93	0,99	1,0

As can be seen from Table 3, the tests power slightly increases with the growth of random effect magnitude. Moreover, it can be seen from Tables 2 and 3, that VERP test is a bit more powerful than the LR test in the considered cases.

4. The GaAs lasers data analysis using LR and VERP tests

In this section, we illustrate the analysis of the GaAs lasers data [10, 11] with the use of proposed LR and VERP tests. Gallium arsenide (GaAs) lasers are used in telecommunication systems, processing of materials, various fields of medicine. The aging process of some lasers leads to deterioration of light output throughout the whole life cycle. The lasers fail when the consumption current exceeds nominal value on 10%. Developing the lasers, engineers had some requirements: lasers have to work no less than 200000 hours under temperature of 20°C without failure. During the accelerated experiment 15 lasers were tested under the stress of 80°C for 40 000 hours. The degradation paths for tested lasers are shown in Figure 5.

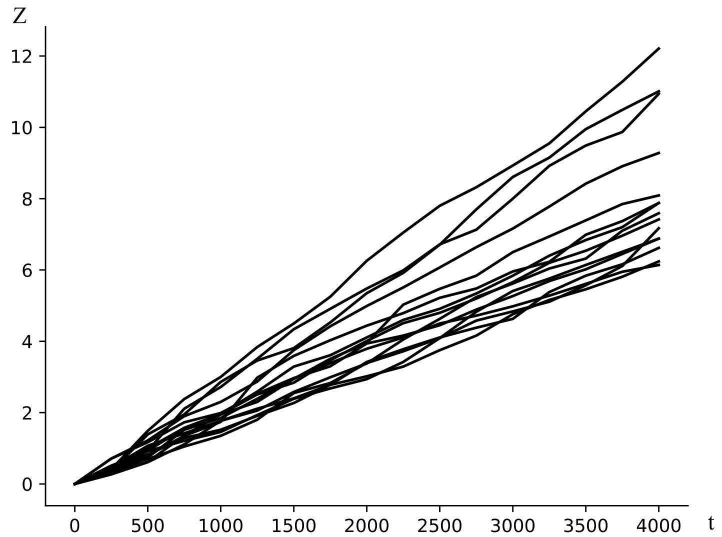

Fig. 5. The degradation paths for the GaAs lasers example

As can be seen from Figure 5, the degradation paths distinctly differ from each other. However, we cannot be sure that the random effect is significant here. Thereby, it is necessary to test the hypothesis of absence of random effect using proposed tests.

The results of the model parameters estimation, test statistics values and corresponding *p*-values for LR and VERP tests are presented in Table 4.

Table 4

MLEs of gamma degradation model parameters, test statistics values and *p*-values for LR and VERP tests

Gamma degradation model	MLEs of model parameters	LR test		VERP test	
		λ_n	*p*-value	d_n	*p*-value
"Fixed-effect" model	$\hat{\sigma}_n = 14.15$, $\hat{\gamma}_n = 0.002$	24,24	$< 10^{-4}$	0,0001	$< 10^{-4}$
"Random-effect" model	$\hat{\delta}_n = 1{,}45$, $\hat{\theta}_n = 28{,}86$, $\hat{\gamma}_n = 0{,}002$				

Considering the fact that *p*-value < $\alpha = 0{,}05$ for both LR and VERP test, the hypothesis of absence of random effect is rejected. Therefore, the "random-effect" gamma degradation model is more appropriate model for the GaAs lasers data.

In Figure 6, the reliability functions of the "fixed-effect" and "random-effect" gamma degradation models (dashed and solid line correspondently) and the empirical reliability function of the interpolated lasers failures are presented. As can be seen from the figure, the reliability function of the "random-effect" model is closer to the observed failure distribution. So, this fact demonstrates that the gamma degradation model with random effects is more appropriate for describing considered GaAs lasers data.

Conclusion

In this paper, we have considered the problems of testing the hypothesis of absence of random effects in degradation data. The likelihood ratio test (LR test) and based of the variance estimate of the random parameter σ (VERP test) were proposed to reveal the existence of random effects in degradation data corresponding to the gamma degradation model. The conducted research of the tests power showed that the VERP test is a bit more powerful criteria than the LR test for smaller sample sizes.

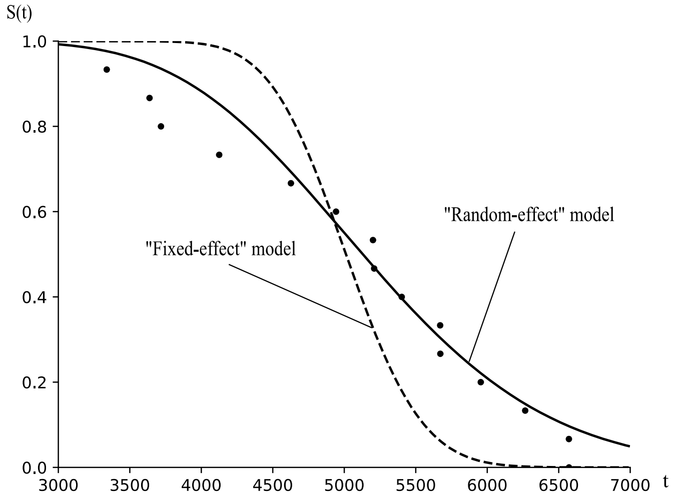

Fig. 6. The reliability functions of the “fixed-effect” and “random-effect” gamma degradation models and the empirical distribution of lasers failures

The example with the GaAs lasers data was considered. Based on the results of the investigations, we recommend to use the gamma degradation model with random effects for the further analysis of the lasers data because this model is more appropriate for describing the change of degradation index than the “fixed-effect” model.

REFERENCES

1. Nikulin, M. & Bagdonavicius, V. (2001) *Accelerated Life Models: Modeling and Statistical Analysis*. Boca Raton: Chapman & Hall/CRC.
2. Antonov, A.V. & Nikulin, M.S. (2012) *Statisticheskie modeli v teorii nadezhnosti* [Statistical models in reliability theory]. Moscow: Abris.
3. Lawless, J. & Crowder, M. (2004) Covariates and Random Effects in a Gamma Process Model with Application to Degradation and Failure. *Life Data Analysis.* 10. pp. 213–227. DOI: 10.1023/B:LIDA.0000036389.14073.dd.
4. Tsai, C.-C., Tseng, S.-T. & Balakrishnan, N. (2012) Optimal Design for Degradation Tests Based on Gamma Processes with Random Effects. *IEEE Trans. Reliab.* 61. pp. 604–613. DOI: 10.1109/TR.2012.2194351
5. Chimitova, E.V. & Chetvertakova, E.S. (2015) A comparison of the “fixed-effect” and “random-effect” gamma degradation models. *Applied methods of statistical analysis. Nonparametric approach, AMSA’2015, September 14–19, 2015: Proc. of the Int. Workshop*. Novosibirsk. pp. 161–169.
6. Tsai, C.-C., Tseng, S.-T. & Balakrishnan, N. (2011) Mis-specification analyses of gamma and Wiener degradation processes. *Journal of Statistical Planning and Inference*. 12. pp. 25–35. DOI: 10.1016/j.jspi.2011.06.008
7. Chimitova, E. & Chetvertakova, E. (2014) The construction of the gamma degradation model with covariates. *Vestnik Tomskogo gosudarstvennogo universiteta. Upravlenie, vychislitel'naya tekhnika i informatika – Tomsk State University Journal of Control and Computer Science*. 4(29). pp. 51–60.
8. Hausman, J.A. (1978) Specification Tests in Econometrics. *Econometrica*. 46. pp. 1251–1271.
9. Chimitova, E., Chetvertakova, E., Sergeeva, S. & Osintseva, E. (2017) A comparative analysis of the Wiener, Gamma and Inverse Gaussian degradation models. *Applied Methods of Statistical Analysis. Nonparametric methods in cybernetics and system analysis. Krasnoyarsk, Russia, September 18–22, 2017: Proc. of the Int. Workshop.* Novosibirsk: NSTU. pp. 160–167.
10. Meeker, W.Q. & Escobar, L.A. (1998) *Statistical Methods for Reliability Data*. New York: John Wiley and Sons.
11. Meeker, W.Q., Doganaksoy, N. & Hahn, G.J. (2009) Ispolzovanie dannykh o degradatsii dlya analiza nadezhnosti izdeliy [Using degradation data to analyze product reliability]. *Metody menedzhmenta kachestva – Methods of Quality Management*. 4.

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Деградационные гамма-модели широко используются для оценки функции надежности по данным об изменении показателя деградации во времени. В данной статье рассматриваются проблемы построения деградационной гамма-модели со случайным эффектом, которая учитывает разброс между деградационными процессами. Предложены два статистических критерия, которые позволяют выявить наличие случайного эффекта в данных, соответствующих рассматриваемой модели. Первый критерий представляет собой хорошо известный критерий отношения правдоподобия, а второй основан на оценке дисперсии случайного параметра. С использованием методов имитационного моделирования проведено исследование мощности данных критериев. Применение разработанных критериев рассмотрено на примере данных об исследовании арсенид-галлиевых (GaAs) лазеров.

Ключевые слова: деградационная гамма-модель; модель с фиксированным эффектом; модель со случайным эффектом; надежность; арсенид-галлиевые лазеры.

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