ВЕСТНИК ТОМСКОГО ГОСУДАРСТВЕННОГО УНИВЕРСИТЕТА
2020 Управление, вычислительная техника и информатика № 50

УДК 519.24
DOI: 10.17223/19988605/50/8

V.M. Chubich, E.V. Filippova

ACTIVE PARAMETRICAL IDENTIFICATION OF STOCHASTIC LINEAR CONTINUOUS-DISCRETE SYSTEMS BASED ON THE EXPERIMENT DESIGN IN THE PRESENCE OF ABNORMAL OBSERVATIONS

The reported study was funded by RFBR according to the research project No. 18-31-00283.

The procedure of active parametrical identification of stochastic linear continuous-discrete systems including robust estimation of parameters and optimal design of input signals is offered. A general case of entering unknown parameters into the equations of state and observation, initial conditions and covariance matrices of system noise and measurements is considered. The efficiency of this procedure is demonstrated by the example of a direct current motor control system.
Keywords: continuous-discrete system; anomalous observations; optimal input signal; robust estimation.

Development of information technologies for identification of complex dynamic systems of stochastic nature is an important area of research and has attracted considerable interest. Application of the optimal experiment theory methods in parametrical identification improves the quality of the results by taking into account more fully the properties of the dynamic object and data collection procedures [1–7]. Thus, given the structure of the mathematical model, the procedure of active parametrical identification involves the following steps:

– Calculation of parameter estimates based on measurement data corresponding to a test input signal.

– Synthesis based on the obtained estimates of the optimal input signal (experiment design).

– Recalculation of estimates of unknown parameters according to the measured data corresponding to the synthesized signal.

Traditionally, the estimation of unknown parameters is carried out based on the classical Kalman filter, which makes it possible to find estimates of the state vector and corresponding covariance matrices under the assumption of the normality of the noise distribution of the system and measurements. When solving practical problems (for example, problems of communication, navigation, control and radar) there are cases when the usual mechanism of formation of observation data is broken and the appearance of anomalous observations that do not contain information about the object under study. In this case the specified filter can lead to biased estimates or even diverge. At the moment, numerous robust modifications of the Kalman filter, resistant to the appearance of outliers, have been developed. In this regard, it is advisable to consider robust estimation methods that provide good quality results.

This work is devoted to the development of mathematical and software procedures of active identification of stochastic continuous-discrete systems based on robust parameter estimation. The efficiency of the developed procedure is demonstrated by the example of one model structure.

1. Problem statement

Consider the following controlled, observed, identifiable dynamic system model in state space:

$$\frac{d}{dt}x(t) = F(t)x(t) + \Psi(t)u(t) + \Gamma(t)w(t),\ t \in [t_0, t_N], \tag{1}$$

$$y(t_{k+1}) = H(t_{k+1})x(t_{k+1}) + v(t_{k+1}),\ k = 0,1,...,N-1, \tag{2}$$

where $x(t)$ is the state *n*-vector; $u(t)$ is deterministic control (input) *r*-vector; $w(t)$ is the process noise *p*-vector; $y(t_{k+1})$ is the measurement (output) *m*-vector; $\nu(t_{k+1})$ is the measurement error *m*-vector.

Let us suppose that

– the random vectors $w(t)$ and $\nu(t_{k+1})$ form a white Gaussian noise, for which

$$\mathrm{E}\left[w(t)\right]=0, \quad \mathrm{E}\left[w(t)w^{\mathrm{T}}(\tau)\right]=Q(t)\delta(t-\tau),$$

$$\mathrm{E}\left[v(t_{k+1})\right]=0, \ \mathrm{E}\left[v(t_{k+1})v^{\mathrm{T}}(t_{i+1})\right]=R(t_{k+1})\delta_{ki},$$

$$\mathrm{E}\left[\nu(t_{k+1})w^{\mathrm{T}}(\tau)\right]=0$$

(here $\mathrm{E}[\ ]$ is operator of mathematical expectation, $\delta(t-\tau)$ is delta function, δ_{ki} is the Kronecker symbol);

– initial state $x(t_0)$ has a normal distribution with parameters

$$\mathrm{E}\left[x(t_0)\right]=\bar{x}(t_0), \quad \mathrm{E}\left\{\left[x(t_0)-\bar{x}(t_0)\right]\left[x(t_0)-\bar{x}(t_0)\right]^{\mathrm{T}}\right\}=\mathrm{P}(t_0)$$

and is uncorrelated with $w(t)$ and $\nu(t_{k+1})$ for all values of k;

– output data may contain outliers;

– unknown parameters are summarized in the *s*-vector θ, including the elements of matrices $F(t)$, $\Psi(t)$, $\Gamma(t)$, $H(t_{k+1})$, $Q(t)$, $R(t_{k+1})$, $\mathrm{P}(t_0)$ and vector $\bar{x}(t_0)$ in various combinations.

For the mathematical model (1), (2), taking into account the a priori assumptions, it is necessary to develop procedures for the active parametrical identification of stochastic continuous-discrete systems based on robust parameter estimation and conduct a numerical study of the effectiveness of its application.

2. Methods of research

Let us consider the main theoretical aspects of the active identification procedure.

Parameter estimation. Unknown parameters estimations of the mathematical model (1), (2) are carried out according to observational data Ξ by using some criterion of identification χ. The collection of numerical data occurs during identification experiments which are carried out under some discrete design ξ_ν:

$$\xi_\nu=\begin{Bmatrix} u^1(t),u^2(t),\dots,u^q(t) \\ \dfrac{k_1}{\nu}\ , \ \dfrac{k_2}{\nu}\ \ ,\dots, \ \dfrac{k_q}{\nu} \end{Bmatrix}, \ u^i(t)\in\Omega_u, \ \ i=1,\dots,q.$$

Here ν is the total number of launches of the system, q is the number of points of the design, k_i is the number of experiments corresponding to the signal $u^i(t)$, Ω_u is the set of design (determined by restrictions on the conditions of the experiment).

Let us denote through $Y_{ij}^{\mathrm{T}}=\left[\left[y^{ij}(t_1)\right]^{\mathrm{T}},\dots,\left[y^{ij}(t_N)\right]^{\mathrm{T}}\right]$ realization of the output signal with number j ($j=1,\dots,k_i$) corresponding to the input signal $u^i(t)$. Then

$$\Xi=\left\{\left(u^i(t),Y_{ij}\right), \ j=1,2,\dots,k_i, \ i=1,2,\dots,q\right\}, \ \sum_{i=1}^{q}k_i=\nu.$$

Due to the fact that the measurement data contain anomalous observations, we will calculate quasi-likelihood estimates [8], solving the following optimization problem:

$$\hat{\theta}=\arg\min_{\theta\in\Omega_\theta}\left[\chi(\theta;\Xi)\right]=\arg\min_{\theta\in\Omega_\theta}\left[-\ln L(\theta;\Xi)\right]. \tag{3}$$

Here

$$\chi(\theta;\Xi) = \frac{Nm\nu}{2}\ln 2\pi + \frac{1}{2}\sum_{i=1}^{q}\sum_{j=1}^{k_i}\sum_{k=0}^{N-1}\left[\varepsilon^{ij}(t_{k+1})\right]^T\left[B^i(t_{k+1})\right]^{-1}\left[\varepsilon^{ij}(t_{k+1})\right] +$$

$$+\frac{1}{2}\sum_{i=1}^{q}k_i\sum_{k=0}^{N-1}\ln\det B^i(t_{k+1}), \tag{4}$$

where $\varepsilon^{ij}(t_{k+1})$ and $B^i(t_{k+1})$ determined by the recurrent equations of a robust filter.

The calculation of the conditional minimum (3) will be carried out by the method of sequential quadratic programming [9, 10], implemented in the optimization Toolbox MATLAB package and assuming the calculation of the gradient.

Differentiating the equality (4) by θ_α $(\alpha = 1,...,s)$ taking into account expression

$$\frac{\partial \ln\det B(t_{k+1})}{\partial\theta_\alpha} = Sp\left[B^{-1}(t_{k+1})\frac{\partial B(t_{k+1})}{\partial\theta_\alpha}\right];$$

$$\frac{\partial B^{-1}(t_{k+1})}{\partial\theta_\alpha} = -B^{-1}(t_{k+1})\frac{\partial B(t_{k+1})}{\partial\theta_\alpha}B^{-1}(t_{k+1})$$

and symmetry of the matrix $B(t_{k+1})$, we obtain

$$\frac{\partial\chi(\theta;\Xi)}{\partial\theta_\alpha} = \sum_{i=1}^{q}\sum_{j=1}^{k_i}\sum_{k=0}^{N-1}\left\{\left[\frac{\partial\varepsilon^{ij}(t_{k+1})}{\partial\theta_\alpha}\right]^{\mathrm{T}}\left[B^i(t_{k+1})\right]^{-1}\left[\varepsilon^{ij}(t_{k+1})\right] -\right.$$

$$\left.-\frac{1}{2}\left[\varepsilon^{ij}(t_{k+1})\right]^T\left[B^i(t_{k+1})\right]^{-1}\frac{\partial B^i(t_{k+1})}{\partial\theta_\alpha}\left[B^i(t_{k+1})\right]^{-1}\left[\varepsilon^{ij}(t_{k+1})\right]\right\} +$$

$$+\frac{1}{2}\sum_{i=1}^{q}k_i\sum_{k=0}^{N-1}Sp\left\{\left[B^i(t_{k+1})\right]^{-1}\frac{\partial B^i(t_{k+1})}{\partial\theta_\alpha}\right\}.$$

Derivatives of $\dfrac{\partial\varepsilon^{ij}(t_{k+1})}{\partial\theta_\alpha}$ and $\dfrac{\partial B^i(t_{k+1})}{\partial\theta_\alpha}$ determined by the equations arising from the corresponding relations of the robust filter.

In [11] the authors conducted a comparative analysis of the efficiency of some modern robust filters for non-stationary linear continuous-discrete systems. While best and quite comparable the results showed correntropy filters Izanloo–Fakoorian–Yazdi–Simon [12] and Chen–Liu–Zhao-Principe [13]. From the point of view of the organization of calculations and, as a consequence, the software implementation of the first of these filters is much easier. In this regard, it seems appropriate to use the Izanloo–Fakoorian–Yazdi–Simon filter when estimating the parameters of models of stochastic linear continuous-discrete systems in the presence of anomalous observations. The corresponding recurrence relations for a single system startup are shown below.

Izanloo–Fakoorian–Yazdi–Simon filter. *Initialization*:

$$\hat{x}(t_0\,|\,t_0) = \bar{x}(t_0),\ P(t_0\,|\,t_0) = P(t_0);\ \sigma = \sigma_0.$$

To run in a loop on $k = \overline{0, N-1}$*:*

$$\frac{d}{dt}\hat{x}(t\,|\,t_k) = F(t)\hat{x}(t\,|\,t_k) + \Psi(t)u(t),\ t\in[t_k, t_{k+1}];$$

$$\frac{d}{dt}P(t\,|\,t_k) = F(t)P(t\,|\,t_k) + P(t\,|\,t_k)F^T(t) + \Gamma(t)Q(t)\Gamma^T(t), t\in[t_k, t_{k+1}];$$

$$\varepsilon(t_{k+1}) = y(t_{k+1}) - H(t_{k+1})\hat{x}(t_{k+1}\,|\,t_k);\quad L(t_{k+1}) = \exp\left(-\frac{\varepsilon^{\mathrm{T}}(t_{k+1})R^{-1}(t_{k+1})\varepsilon(t_{k+1})}{2\sigma^2}\right);$$

$$B(t_{k+1}) = H(t_{k+1})P(t_{k+1}|t_k)L(t_{k+1})H^T(t_{k+1}) + R(t_{k+1});\ K(t_{k+1}) = P(t_{k+1}|t_k)L(t_{k+1})H^{\mathrm{T}}(t_{k+1})B^{-1}(t_{k+1});$$
$$\hat{x}(t_{k+1}|t_{k+1}) = \hat{x}(t_{k+1}|t_k) + K(t_{k+1})\varepsilon(t_{k+1});\ P(t_{k+1}|t_{k+1}) = \left[\mathrm{I} - K(t_{k+1})H(t_{k+1})\right]P(t_{k+1}|t_k).$$

End of loop.

Algorithms for calculating the maximum likelihood criterion and its gradient based on robust filtering for linear continuous-discrete models are presented in [14].

Experiment design. Let us consider the features of input signal design for models of continuous-discrete systems (1), (2). Continuous normalized design in this case can be specified as

$$\xi = \begin{Bmatrix} u^1(t), u^2(t), \dots, u^q(t) \\ p_1 \ \ ,\ \ p_2 \ \ ,\dots,\ \ p_q \end{Bmatrix},\ p_i \ge 0, \sum_{i=1}^{q} p_i = 1, u^i(t) \in \Omega_u,\ i = 1, 2, ..., q. \tag{5}$$

Unlike discrete design ξ_ν, weights p_i in continuous design ξ can take any values, including irrational number.

Information matrix $M(\xi)$ for design (5) is determined by the relation

$$M(\xi) = \sum_{i=1}^{q} p_i M\left(u^i(t), \hat{\theta}\right),$$

in which the information matrices of single-point design depend on the unknown parameters to be estimated and are calculated in accordance with [15].

We find the optimal experiment design for some convex functional X information matrix $M(\xi)$ by solving the following extremal problem

$$\xi^* = \arg\min_{\xi \in \Omega_\xi} \mathrm{X}\left[M(\xi)\right]. \tag{6}$$

The construction of optimal design can be associated with the representation of the components of the input signals in the form of linear combinations of basic functions (as such, you can use orthogonal polynomials Legendre, Chebyshev, Walsh function, etc.) and then search for the coefficients of such linear combinations. Another approach is related to the assumption that the input signals are piecewise-constant functions preserving their values on the interval between adjacent measurements. In [16] have demonstrated the effectiveness and applicability of the piecewise-constant approximation of the input signal, which makes it possible to calculate the derivatives of the information matrix Fisher from the components of the input signal by recurrent analytical formulas and, consequently, to apply gradient procedures for the synthesis of optimal signals. This means that

$$u^i(t) = \left[u^i(t_0), u^i(t_1), ..., u^i(t_{N-1})\right]^{\mathrm{T}} = U_i.$$

Based on the method of sequential quadratic programming, we present a combined procedure for constructing D- or A-optimal continuous design for pre-computed estimates of the parameters $\hat{\theta}$, which using direct and dual approaches [17–19] for solving the extremal problem (6).

1. Set the initial nondegenerate design

$$\xi_0 = \begin{Bmatrix} U_1^0, U_2^0, ..., U_q^0, \\ p_1^0,\ p_2^0, ...,\ p_q^0 \end{Bmatrix},\quad U_i^0 \in \Omega_u,\quad p_i^0 = \frac{1}{q},\quad i = 1, 2, ..., q.$$

Calculate the information matrix of a single-point designs $M\left(U_i^0, \hat{\theta}\right)$ for $i = 1, ..., q$ and put $k = 0$.

2. Counting the design weight $p_1^k, ..., p_q^k$ are fixed, find the design

$$\bar{\xi}_{k+1} = \arg\min_{U_1^k, ..., U_q^k \in \Omega_u} \mathrm{X}\left[M(\xi_k)\right].$$

Calculate the information matrix of a single-point designs $M\left(U_i^{k+1}, \hat{\theta}\right)$, $i = 1, ..., q$.

3. Having fixed the points of the design spectrum $\overline{\xi}_{k+1}$, we find the design

$$\xi_{k+1} = \arg \min_{p_1^k,\ldots,p_q^k} X\left[M\left(\overline{\xi}_{k+1} \right) \right],\ p_i^k \ge 0,\ \sum_{i=1}^{q} p_i^k = 1,\ i = 1,\ldots,q.$$

4. If an inequality holds for a small positive δ_1

$$\sum_{i=1}^{q} \left[\left\| U_i^{k+1} - U_i^k \right\|^2 + \left(p_i^{k+1} - p_i^k \right)^2 \right] \le \delta_1 ,$$

then let's put $\xi_0 = \xi_{k+1}$ (the execution of the direct procedure is finished), $k = 0$ and go to step 5. Otherwise, take $k = k+1$ and go to step 2.

5. Calculate the information matrix $M\left(\xi_k\right)$.

6. Find the local maximum

$$U^k = \arg \max_{U \in \Omega_u} \mu(U,\ \xi_k)\ .$$

If the condition $\left| \mu\left(U^k,\ \xi_k\right) - \eta \right| \le \delta_2$ for a small positive δ_2 is met, then the process is finished. If $\mu\left(U^k,\ \xi_k\right) > \eta,$ let's move on the step 7, otherwise, to seek a new local maximum.

7. Find τ_k

$$\tau_k = \arg \min_{0 \le \tau \le 1} X\left[M\left(\xi_{k+1}^{\tau} \right) \right].$$

Here $\xi_{k+1}^{\tau} = (1-\tau)\xi_k + \tau\xi(U^k)$, where $\xi(U^k)$ is a one-point design posted at the point U^k.

8. Make a design

$$\xi_{k+1} = (1-\tau_k)\xi_k + \tau_k \xi(U^k),$$

let's clean it up by following [17], put $k = k+1$ and go to step 5.

The correspondence of the parameters $X\left[M\left(\xi\right)\right]$, $\mu(U,\ \xi)$, η of the combined procedure to the specified criteria is presented in table 1.

Table 1

Parameters of the combined procedure

Criterion	Parameters		
	$X\left[M\left(\xi\right)\right]$	$\mu(U,\ \xi)$	η
D	$-\ln \det M(\xi)$	$\mathrm{Sp}\left[M^{-1}(\xi)M(U)\right]$	s
A	$\mathrm{Sp}\left[M^{-1}(\xi)\right]$	$\mathrm{Sp}\left[M^{-2}(\xi)M(U)\right]$	$\mathrm{Sp}\left[M^{-1}(\xi)\right]$

The analytical expressions of the required for the combined procedure derivatives and its calculation algorithms are presented in [15].

Practical application of the synthesized optimal design is difficult, because weights are arbitrary real numbers, enclosed in the interval from zero to one. In the case of a given number ν of possible system starts, it is necessary to "round" the continuous design to discrete [18]. As a result, we obtain a discrete design

$$\xi_\nu = \left\{ \begin{array}{l} U_1^*, U_2^*, \ldots, U_q^* \\ \dfrac{k_1^*}{\nu}, \dfrac{k_2^*}{\nu}, \ldots, \dfrac{k_q^*}{\nu} \end{array} \right\},$$

we perform identification experiments and recalculate estimates of unknown parameters.

3. Simulation

We assume that all a priori assumptions made in the problem definition are fulfilled. Following [20], consider a position control system consisting of an antenna and a direct current (DC) motor. Let the first component of the state vector be responsible for the angular position of the antenna, the second - for its angular velocity. The input signal is the voltage at the input of the DC amplifier controlling the motor. The angular position is measured using a potentiometer. Then the models of state and observation can be determined by the relations:

$$\frac{d}{dt}x(t)=\begin{bmatrix}0 & 1\\ 0 & -\theta_1\end{bmatrix}x(t)+\begin{bmatrix}0\\ \theta_2\end{bmatrix}u(t)+\begin{bmatrix}0\\ 1\end{bmatrix}w(t),\ \ t\in[0,30];$$

$$y(t_{k+1})=[1\quad 0]\,x(t_{k+1})+v(t_{k+1}),\quad k=0,\ldots,N-1.$$

Here θ_1,θ_2 are unknown parameters and $\Omega_\theta=\{1<\theta_1<10,\ 0<\theta_2<1\}$.

Set $N=30$, $u(t)=12$, $Q(t)=Q=0{,}01$, $R(t_{k+1})=R=0{,}1$, $\bar{x}(t_0)=\begin{bmatrix}0\\ 0\end{bmatrix}$, $P(t_0)=\begin{bmatrix}0{,}01 & 0\\ 0 & 0{,}01\end{bmatrix}$. Assume that the measurements are made uniformly every single second and the value of the parameter Izanloo–Fakoorian–Yazdi–Simon filter is $\sigma=10$. Chose area of allowable values of input signals

$$\Omega_u=\{2\le u(t)\le 30,\ t\in[0;30]\}.$$

We simulate samples with anomalous observations using the MATLAB software environment by setting the pollution coefficient of the sample $\lambda=0.1$ and the noise dispersion of anomalous observations $R_A=1000R$, assuming that the true values of the parameters $\theta_1^*=4.600$, $\theta_2^*=0.787$.

To reduce the dependence of the estimation results on the experimental data, we perform five independent starts of the system and average the obtained estimates of the unknown parameters. The quality of parametric identification will be judged by the value of the relative estimation error δ_θ, calculated by formula:

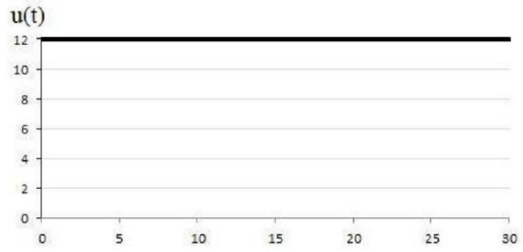
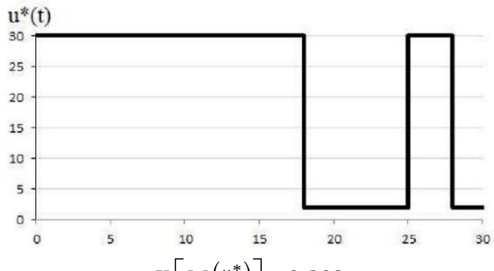
$$\delta_\theta=\sqrt{\frac{\left(\theta_1^*-\hat{\theta}_1\right)^2+\left(\theta_2^*-\hat{\theta}_2\right)^2}{\left(\theta_1^*\right)^2+\left(\theta_2^*\right)^2}},$$

where $\hat{\theta}=\left(\hat{\theta}_1,\hat{\theta}_2\right)$ is vector of unknown parameters estimates.

Numerical results of calculations are presented in table 2 (the optimal design is one-point).

Table 2

Results of robust procedure of active parametrical identification

Input signal and values of the D-optimality criterion	System start number	Estimates and estimation errors		
		$\hat{\theta}_1$	$\hat{\theta}_2$	δ_θ
	1	5,012	0,547	0,102
	2	3,411	0,689	0,255
	3	3,632	0,302	0,232
	4	4,890	0,705	0,064
	5	3,075	0,488	0,332
$X[M(\xi)]=0{,}791$	Average value for startups	4,004	0,546	0,137
	1	4,277	0,705	0,071
	2	4,332	0,791	0,057
	3	5,016	0,589	0,098
	4	4,170	0,691	0,094
	5	3,994	0,722	0,130
$X[M(\xi^*)]=0{,}208$	Average value for startups	4,357	0,699	0,055

Analysis of table 2 contents shows that design of D-optimal input signal using the combined procedure make it possible to improve the quality of estimation by 8,2%.

Thus, the authors consider that the applying of the active parametrical identification procedures based on robust estimation and optimal design of input signals is helpful and advisable in the presence of outliers in the measurement data.

Conclusion

Robust procedure of active parametrical identification for models of stochastic linear continuous-discrete systems including robust estimation of parameters based on the Izanloo–Fakoorian–Yazdi–Simon filter and optimal design of input signal is developed. The case of the entry of unknown parameters into the state and observations equations, the initial conditions and the covariance noise matrices of the system and measurements is considered. The efficiency of the developed robust procedure of active parametrical identification is demonstrated on the example of a DC motor control system for the random nature of the outliers.

REFERENCES

1. Ljung, L. (1987) *System identification: Theory for the User*. Englewood Cliffs, NJ: Prentice-Hall.
2. Walter, E. & Pronzato, L. (1997) *Identification of parametric models from experimental data*. Berlin: Springer-Verlag.
3. Jauberthie, C., Bournonville, F., Coton, P. & Rendell, F. (2006) Optimal input design for aircraft parameter estimation. *Aerospace Science and Technology*. 10. pp. 331–337. DOI:10.1016/j.ast.2005.08.002
4. Brasil, N., Hemerly, E. & Goes, L. (2009) Aircraft parameter estimation experiment design considering measurement colored residuals. *Journal of Aircraft*. 46(6). pp. 1857–1865. DOI:10.2514/1.34133
5. Bernaerts, K., Servaes, R.D., Kooyman, S., Versyck, K.J. & Van Impe, J. (2002) Optimal temperature input design for estimation of the Square Root model parameters: parameter accuracy and model validity restrictions. *International Journal of Food Microbiology*. 73. pp. 145–157. DOI: 10.1016/S0168-1605(01)00645-6
6. Galvanin, F., Marchesini, R., Barolo, M., Bezzo, F. & Fidaleo, M. (2016) Optimal design of experiments or parameter identification in electrodialysis models. *Chemical Engineering Research and Design*. 105. pp. 107–119. DOI: 10.1016/j.cherd.2015.10.048
7. Chubich, V.M. & Filippova, E.V. (2017) Information technology of active identification of stochastic dynamic systems using parameterization of the input signal. *Evraziyskoe Nauchnoe Ob"edinenie – Eurasian Scientific Association*. 11(33). pp. 63–66.
8. Mudrov, V.I. & Kushko, V.L. (1983) *Metody obrabotki izmereniy: Kvazipravdopodobnye otsenki* [Methods of measurement processing. Quasi-likelihood estimates]. Moscow: Radio i svyaz'.
9. Antoniou, A. & Lu, W-S. (2007) *Practical optimization: algorithms and engineering applications*. New York: Springer. DOI:10.1007/978-0-387-71107-2
10. Izmailov, A.F. & Solodov, M.V. (2008) *Chislennye metody optimizatsii* [Numerical optimization methods]. Moscow: Fizmat.
11. Chubich, V.M. & Filippova, E.V. (2018) Research of the efficiency of some robust filters for non-stationary linear continuous-discrete systems. *Sovremennye naukoemkie tekhnologii – Modern High Technologies*. 12(1). pp. 153–161.
12. Izanloo, R., Fakoorian, S.A., Yazdi, H.S. & Simon, D. (2016) Kalman filtering based on the maximum correntropy criterion in the presence of non-Gaussian noise. *Annual Conference on Information Science and Systems (CISS), Princeton, USA*. pp. 500–505. DOI: 10.1109/CISS.2016.7460553
13. Chen, B., Liu, X., Zhao, H. & Principe, J. (2017) Maximum correntropy Kalman filter. *Automatica*. 76. pp. 70–77. DOI: 10.1016/j.automatica.2016.10.004
14. Filippova, E.V. (2018) Algorithms of parametrical identification of stochastic linear continuous-discrete systems based on robust filtering. *Evraziyskoe Nauchnoe Ob"edinenie – Eurasian Scientific Association*. 11(45). pp. 86–90.
15. Chubich, V.M. & Filippova, E.V. (2017) *Aktivnaya identifikatsiya stokhasticheskikh dinamicheskikh sistem. Planirovanie eksperimenta dlya modeley diskretnykh sistem* [Active identification of stochastic dynamic systems. Design of experiments for models of discrete systems]. Novosibirsk: NSTU.
16. Denisov, V.I., Chubich, V.M. & Filippova, E.V. (2018) The choice of the parameterization of the input signal in the problem of experiment design for model stochastic systems. *Izvestiya Tul'skogo gosudarstvennogo universiteta. Tekhnicheskie nauki – Proc. of Tula State University. Technical Science*. 2. pp. 387–397.
17. Fedorov, V.V. (1971) *Teoriya optimal'nogo eksperimenta* [Theory of optimal experiment]. Moscow: Nauka.
18. Ermakov, S.M. & Zhiglyavsky, A.A. (1987) *Matematicheskaya teoriya optimal'nogo eksperimenta* [Mathematical theory of optimal experiment]. Moscow: Nauka.

19. Pronzato, L. & Pazman, A. (2013) *Design of Experiments in Nonlinear Models. Asymptotic Normality, Optimality Criteria and Small-Sample Properties*. New York: Springer. DOI: 10.1007/978-1-4614-6363-4
20. Kwakernaak, H. & Sivan, R. (1972) *Linear Optimal Control Systems*. New York: John Wiley Sons. DOI: 10.1115/1.3426828

Received: April 5, 2019

Chubich V.M., Filippova E.V. (2020) ACTIVE PARAMETRICAL IDENTIFICATION OF STOCHASTIC LINEAR CONTINUOUS-DISCRETE SYSTEMS BASED ON THE EXPERIMENT DESIGN IN THE PRESENCE OF ABNORMAL OBSERVATIONS. *Vestnik Tomskogo gosudarstvennogo universiteta. Upravlenie vychislitelnaja tehnika i informatika* [Tomsk State University Journal of Control and Computer Science]. 50. pp. 61–68

DOI: 10.17223/19988605/50/8

Чубич В.М., Филиппова Е.В. АКТИВНАЯ ПАРАМЕТРИЧЕСКАЯ ИДЕНТИФИКАЦИЯ СТОХАСТИЧЕСКИХ ЛИНЕЙНЫХ НЕРЕРЫВНО-ДИСКРЕТНЫХ СИСТЕМ НА ОСНОВЕ ПЛАНИРОВАНИЯ ЭКСПЕРИМЕНТА ПРИ НАЛИЧИИ АНОМАЛЬНЫХ НАБЛЮДЕНИЙ. *Вестник Томского государственного университета. Управление, вычислительная техника и информатика.* 2019. № 50. С. 61–68

Предложена процедура активной параметрической идентификации стохастических линейных непрерывно-дискретных систем, включающая робастное оценивание параметров и оптимальное планирование входных сигналов. Рассматривается общий случай вхождения неизвестных параметров в уравнения состояния и наблюдения, начальные условия и ковариационные матрицы шумов системы и измерений. Эффективность данной процедуры продемонстрирована на примере системы управления электродвигателем постоянного тока.

Ключевые слова: непрерывно-дискретная система; аномальные наблюдения; оптимальный входной сигнал; робастное оценивание.

CHUBICH Vladimir Mikhailovich (Doctor of Technical sciences, Professor, Head of the department of theoretical and applied informatics, Novosibirsk State Technical University, Russian Federation).
E-mail: chubich@ami.nstu.ru

FILIPPOVA Elena Vladimirovna (Candidate of Technical sciences, Associate professor, Associate professor of theoretical and applied informatics, Novosibirsk State Technical University, Russian Federation).
E-mail: e.filippova@corp.nstu.ru