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ON THE NUMERICAL SOLUTION TO A NON-CLASSICAL PROBLEM OF BENDING AND STABILITY FOR AN ORTHOTROPIC BEAM OF VARIABLE THICKNESS

The mathematical model of the problem of bending of an elastically clamped beam is constructed on the basis of the refined theory of orthotropic plates of variable thickness. To solve the problem in the case of simultaneous action of its own weight and compressive axial forces, a system of differential equations with variable coefficients is obtained. The effects of transverse shear and the effect of reducing compressive force of the support are also taken into account. Passing on to dimensionless quantities, the specific problem for a beam of linearly varying thickness is solved by the collocation method. The stability of the beam is discussed. The critical values of forces are obtained by varying the axial compressive force. Results are presented in both tabular and graphical styles. Based on the results obtained, appropriate conclusions are drawn.

Keywords: *elastically clamped support, bending, transverse shear, stability.*

Introduction

It is known that the structural elements used in various building structures have the form of beams, plates or shells. To clarify the carrying capacity of such thin-walled elements, sometimes it is necessary to solve the problem of bending and stability under the action of axial compressive forces, taking into account its own weight.

In the scientific literature one can find numerous works in which questions of the stress-strain state and stability of such elements are considered in the framework of the classical theory of mechanics (see, e.g., [1]). In view of the present stage in the development of materials science, one can say that, basically, these elements are anisotropic. This led to the need for the mentioned research on refined theories, which take into account those factors that are neglected in the classical theory. The present paper is devoted to the study of this issue.

Problem Formulation

Consider an orthotropic beam of length l, constant width b, and variable thickness h in the right-hand Cartesian coordinate system x, y, z.

The main directions of the anisotropy of the material are parallel to the coordinate axes. The beam is elastically clamped at the two ends and besides of its own weight is also under the action of axial compressive forces T (Fig. 1).

It is taken into account that the elastically clamped support due to friction with an elastic array reduces the external compressive force P. As a result of this, the following force acts on the beam

$$T = \beta P, \qquad \beta < 1. \tag{1}$$

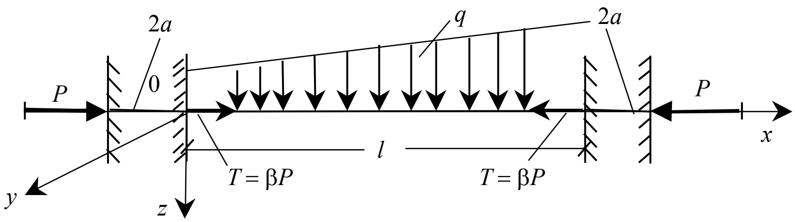

Fig. 1. A beam and compressive forces
Рис. 1. Балка и сжимающая сила

The value of the coefficient β can be easily determined experimentally. It is known [2] that the conditions of the considered elastically clamped support with transverse bending of the beam have the form

$$\frac{dw}{dx} = D\left(aN_x - M_x\right), \quad w = a\frac{dw}{dx} + BN_x . \tag{2}$$

Here, w is the bend; N_x and M_x are the shear force and bending moment of the beam, respectively; D and B are the parameters of the elastically clamped support, which are related by the following relation:

$$D = \frac{3B}{a^2} . \tag{3}$$

The parameters D and B are inverses of the stiffness of the support for rotation and the vertical displacement, respectively. In the system SI they have the following dimensions: $D \sim N^{-1}m^{-1}$, $B \sim m\,N^{-1}$.

Note that in the derivation of conditions (2) it was accepted, due to the relative smallness of the length $2a$, the part of the beam inserted into the elastic array, without deformation, moves progressively and rotates as one part. By virtue of this, the value of the deflection derivative $\frac{dw}{dx}$ do not change within the inserted parts and are equal to the values at $x = 0$ and $x = l$.

Development of a mathematical model of the problem

Using the refined theory of orthotropic plates of variable thickness (see [3], p.18), we obtain the following differential equations for the bending problem of the beam under consideration:

$$\begin{cases} \left(Ebh^2\dfrac{d^2h}{dx^2} + 12\beta P\right)\dfrac{d^2w}{dx^2} - bh\left(8 + \chi h\dfrac{d^2h}{dx^2}\right)\dfrac{d\varphi_1}{dx} - 16b\dfrac{dh}{dx}\varphi_1 = 12\rho gbh, \\ Eh^2\dfrac{d^3w}{dx^3} + 2Eh\dfrac{dh}{dx}\dfrac{d^2w}{dx^2} - \chi h^2\dfrac{d^2\varphi_1}{dx^2} - 2\chi h\dfrac{dh}{dx}\dfrac{d\varphi_1}{dx} + 8\varphi_1 = 0. \end{cases} \tag{4}$$

Due to the absence of stress σ_y and the neglect of stress σ_z, the material parameter B_{11} is replaced by the Young modulus E of the axial direction; χ takes into account the effect of transverse shear deformation e_{xz}; φ_1 is the function that characterizes the distribution of tangential stress τ_{xz} in the mid-plane of the beam $z = 0$.

An axial compressive force acts on the beam, which decreases on the side of the elastic array. Therefore, in the expression of the load term Z_2 [4], to the intensity of the vertical load, arising from its own weight, the intensity of the fictitious load is added, which appears as a result of the compression of a curved beam by forces βP . In system (4), ρ is the density of the beam material and g is the gravity acceleration.

Thus, the problem under consideration is reduced to solving the system of differential equations (4) with boundary conditions (2) written for both edges of the beam: $x=0$ and $x=l$.

The obtained boundary-value problem (4) – (2) is a mathematical model of the problem of beam bending.

Taking into account [3] the shear force of the beam N_x and the bending moment M_x, which differ from the corresponding plate expressions by multiplication by the beam width b , we have

$$\begin{aligned} N_x &= \frac{bh}{12}\left[8\varphi_1 - h\frac{dh}{dx}\left(E\frac{d^2w}{dx^2} - \chi\frac{d\varphi_1}{dx}\right)\right], \\ M_x &= -\frac{bh^3E}{12}\left(\frac{d^2w}{dx^2} - a_{55}\frac{d\varphi_1}{dx}\right). \end{aligned} \tag{5}$$

Solution Technique

For simplicity, we assume that both edges of the beam have the same elastically clamped support.

Let us apply the following dimensionless notations:

$$\begin{gathered} x = l\bar{x},\quad h = m_1 l H,\quad b = m_2 l,\quad a = m_1 l,\quad \varphi_1 = E\bar{\varphi}, \\ P = E m_1^2 l^2 \bar{P},\quad \rho = \frac{E m_1^3 \bar{\rho}}{m_2 g l},\quad D = \frac{3\bar{B}}{E m_1^2 l^3},\quad w = a\bar{w},\quad BEl = \bar{B}. \end{gathered} \tag{6}$$

Consider the case when the thickness of the beam varies linearly:

$$h = m_1 l + h_1 x = m_1 l H \quad \Rightarrow H = 1 + \gamma\bar{x}, \tag{7}$$

where

$$\gamma = \frac{h_1}{m_1},\quad h > 0 \Rightarrow \quad h_1 > -m_1. \tag{8}$$

Taking into account notation (6), equation (4) takes the form

$$\begin{cases} 12m_1^2\beta\bar{P}\dfrac{d^2\bar{w}}{d\bar{x}^2} - 8m_2H\dfrac{d\bar{\varphi}}{d\bar{x}} - 16m_2\dfrac{dH}{d\bar{x}}\bar{\varphi} = 12m_1^3\bar{\rho}H, \\ m_1^3H^2\dfrac{d^3\bar{w}}{d\bar{x}^3} + 2m_1^3H\dfrac{dH}{d\bar{x}}\dfrac{d^2\bar{w}}{d\bar{x}^2} - \chi m_1^2H^2\dfrac{d^2\bar{\varphi}}{d\bar{x}^2} - 2\chi m_1^2H\dfrac{dH}{d\bar{x}}\dfrac{d\bar{\varphi}}{d\bar{x}} + 8\bar{\varphi} = 0. \end{cases} \tag{9}$$

Boundary conditions (2), which must be satisfied at both edges of the beam, in view of (6) and (7), at $\bar{x}=0$ and $\bar{x}=1$, will be

$$\frac{d\overline{w}}{d\overline{x}}=\frac{\overline{B}m_2H}{4m_1}\left[8\overline{\varphi}+m_1H\left(H-m_1\frac{dH}{d\overline{x}}\right)\left(m_1\frac{d^2\overline{w}}{d\overline{x}^2}-\chi\frac{d\overline{\varphi}}{d\overline{x}}\right)\right],$$
$$\overline{w}=m_1\frac{d\overline{w}}{d\overline{x}}+\frac{\overline{B}m_2H}{12}\left[8\overline{\varphi}-m_1^2H\frac{dH}{d\overline{x}}\left(m_1\frac{d^2\overline{w}}{d\overline{x}^2}-\chi\frac{d\overline{\varphi}}{d\overline{x}}\right)\right]. \quad (10)$$

Taking into account (7), equations (9) take the following form:

$$\begin{cases}3\beta m_1^2\overline{P}\dfrac{d^2\overline{w}}{d\overline{x}^2}-2m_2(1+\gamma\overline{x})\dfrac{d\overline{\varphi}}{d\overline{x}}-4m_2\gamma\overline{\varphi}=3m_1^3\overline{\rho}(1+\gamma\overline{x}),\\ m_1^3(1+\gamma\overline{x})^2\dfrac{d^3\overline{w}}{d\overline{x}^3}+2m_1^3\gamma(1+\gamma\overline{x})\dfrac{d^2\overline{w}}{d\overline{x}^2}-\chi m_1^2(1+\gamma\overline{x})^2\dfrac{d^2\overline{\varphi}}{d\overline{x}^2}-2\chi m_1^2\gamma(1+\gamma\overline{x})\dfrac{d\overline{\varphi}}{d\overline{x}}+8\overline{\varphi}=0.\end{cases} \quad (11)$$

And boundary conditions (10) at $\overline{x}=0$ and $\overline{x}=1$, are

$$\frac{d\overline{w}}{d\overline{x}}=\frac{\overline{B}m_2(1+\gamma\overline{x})}{4m_1}\left[8\overline{\varphi}+m_1(1+\gamma\overline{x})(1+\gamma\overline{x}-m_1\gamma)\left(m_1\frac{d^2\overline{w}}{d\overline{x}^2}-\chi\frac{d\overline{\varphi}}{d\overline{x}}\right)\right],$$
$$\overline{w}=m_1\frac{d\overline{w}}{d\overline{x}}+\frac{\overline{B}m_2(1+\gamma\overline{x})}{12}\left[8\overline{\varphi}-m_1^2\gamma(1+\gamma\overline{x})\left(m_1\frac{d^2\overline{w}}{d\overline{x}^2}-\chi\frac{d\overline{\varphi}}{d\overline{x}}\right)\right]. \quad (12)$$

Computational Part

Let us perform calculations for the following values of parameters:

$$m_1=0.1;\ m_2=0.3;\quad \gamma=1;\quad \chi=0,\ 5,\text{ and }10;$$
$$\rho=0.006;\ \beta=0.5;\quad \overline{B}=1. \quad (13)$$

Let us present the unknown functions $\overline{w}$ and $\overline{\varphi}$ as polynomials:

$$\overline{w}=a_0+\sum_{i=1}^{k}a_i\overline{x}^i,\quad \overline{\varphi}=b_0+\sum_{i=1}^{k}b_i\overline{x}^i. \quad (14)$$

The problem is to be solved by the collocation method.

To determine the above coefficients a_0, a_i and b_0, b_i, we divide the interval $0\le\overline{x}\le1$ into k equal parts. To satisfy equations (9) at the dividing points and boundary conditions (10), we obtain a system of $2(k+1)$ linear equations with respect to these coefficients.

After solving the system, we find the values of the mentioned coefficients, with the help of which we calculate the values of functions $\overline{w}$ and $\overline{\varphi}$ using formulas (14).

The dimensionless values of the transverse force $\overline{N}_x$ and the bending moment $\overline{M}_x$ at the end-points and the dividing points of the interval $0\le\overline{x}\le1$, are calculated according the following formulas:

$$\overline{N}_x=\frac{m_1m_2(1+\gamma\overline{x})}{12}\left[8\overline{\varphi}-m_1^2(1+\gamma\overline{x})\gamma\left(m_1\frac{d^2\overline{w}}{d\overline{x}^2}-\chi\frac{d\overline{\varphi}}{d\overline{x}}\right)\right],$$
$$\overline{M}_x=-\frac{m_2m_1^3(1+\gamma\overline{x})^3}{12}\left(m_1\frac{d^2\overline{w}}{d\overline{x}^2}-\chi\frac{d\overline{\varphi}}{d\overline{x}}\right). \quad (15)$$

In this paper, the smallest dimensionless value of the critical force $\overline{P}_{cr}$ is determined by varying the compressive force magnitude until $\overline{w}$ does not change its sign.

The results of the calculations given in Table 1 show that the allowance for the influence of the deformations of transverse shears (case $\chi > 0$) leads to a decrease in the critical force.

Table 1

Critical force values

	variable thickness $(\gamma = 1)$ Density of the material $\overline{\rho} = 0.006$		
	χ $\chi > 0$ (transverse shear is taken into account)		
	0	5	10
$\overline{P}_{cr}$ – critical force	0.45	0.38	0.28

Table 2

Deflection maximum ($\overline{w}_{max} \cdot 10^3$) depending on $\overline{P}/\overline{P}_{cr}$ at $\gamma = 1$ and different values of χ (deflection point is at $\overline{x}_{max} = 0.4$)

	$\overline{P}/\overline{P}_{cr}$										
$\overline{x}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\chi = 0$	0.585	0.634	0.692	0.763	0.853	0.969	1.127	1.353	1.702	2.318	3.689
$\chi = 5$	0.578	0.636	0.707	0.798	0.918	1.081	1.322	1.708	2. 440	4.382	26.42
$\chi = 10$	0.563	0.620	0.689	0.778	0.894	1.052	1.283	1.652	2.341	4.098	18.45

Table 3

Bending values along the beam for $\gamma = 1; \rho = 0.006; \overline{P} = 0.1$

			$\overline{x}$										
			0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
χ	0	$\overline{w} \cdot 10^3$	0.04971	0.2146	0.4601	0.6418	0.7066	0.6551	0.5179	0.3392	0.1674	0.0504	0.0325
	5	$\overline{w} \cdot 10^3$	0.0550	0.2388	0.5074	0.7002	0.7621	0.6984	0.5455	0.3528	0.1727	0.0558	0.0475
	10	$\overline{w} \cdot 10^3$	0.0598	0.2691	0.5705	0.7799	0.8398	0.7614	0.5884	0.3764	0.1829	0.0610	0.0577

Figure 2 shows the changes of the basic quantities depending on the value of the parameter χ at $\overline{P} = 0.2$.

Figure 3 shows the changes of the deflection on the values of the parameters $\overline{P}$ and χ.

Figure 4 shows the changes of the transverse force on the value of the parameters $\overline{P}$ and χ.

Figure 5 shows the changes of the bending moment on the value of the parameters $\overline{P}$ and χ.

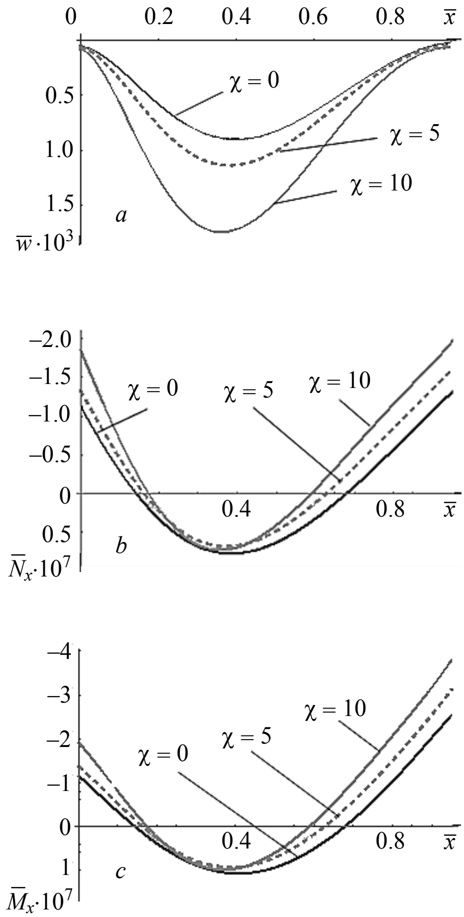

Fig. 2. Distribution of the (*a*) deflection, (*b*) transverse force, and (*c*) bending moment
Рис. 2. Распределение прогиба (*a*), поперечной силы (*b*), изгибающего момента (*c*)

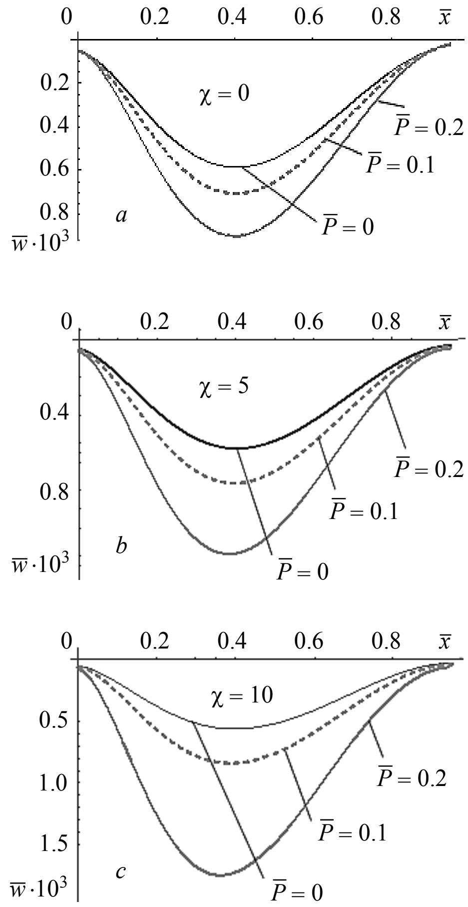

Fig. 3. Distribution of the deflection: $\chi =$ (*a*) 0, (*b*) 5, and (*c*) 10
Рис. 3. Распределение прогиба: $a - \chi = 0$; $b - \chi = 5$; $c - \chi = 10$

Consideration of the effect of transverse shear deformations does not significantly affect the nature of the change in the value of the transverse force and the bending moment (Figs. 2, *b*, *c* and Figs. 4, 5).

Taking into account the effect of transverse shear deformations (case $\chi > 0$), as was expected, with the same values of the other quantities, leads to an increase in deflections (Figs. 3, *a*, *b*, and *c*).

It should be noted that in the scientific literature there are many works devoted to the description and application of the collocation method, as well as the study of the bending and stability of thin-walled elements with different boundary conditions, including the condition of an elastically clamped support [5–28].

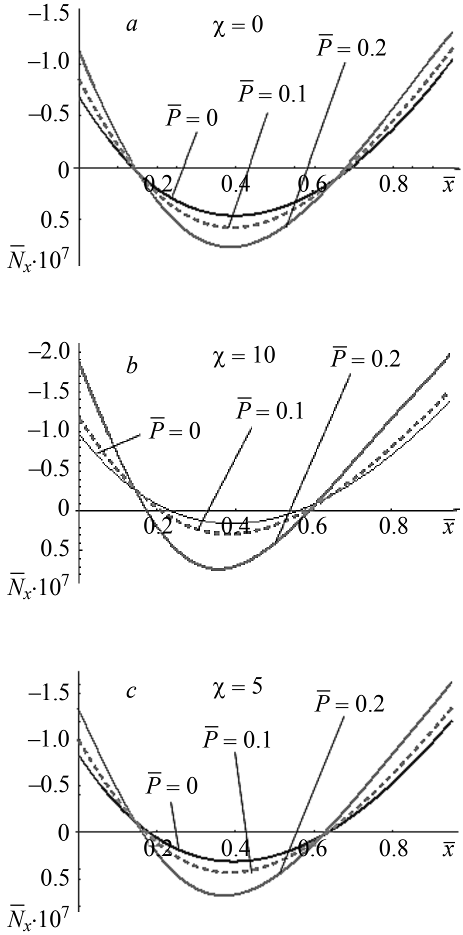

Fig. 4. Distribution of the transverse force: $\chi =$ (*a*) 0, (*b*) 5, and (*c*) 10

Рис. 4. Распределение поперечной силы: $a - \chi = 0$; $b - \chi = 5$; $c - \chi = 10$

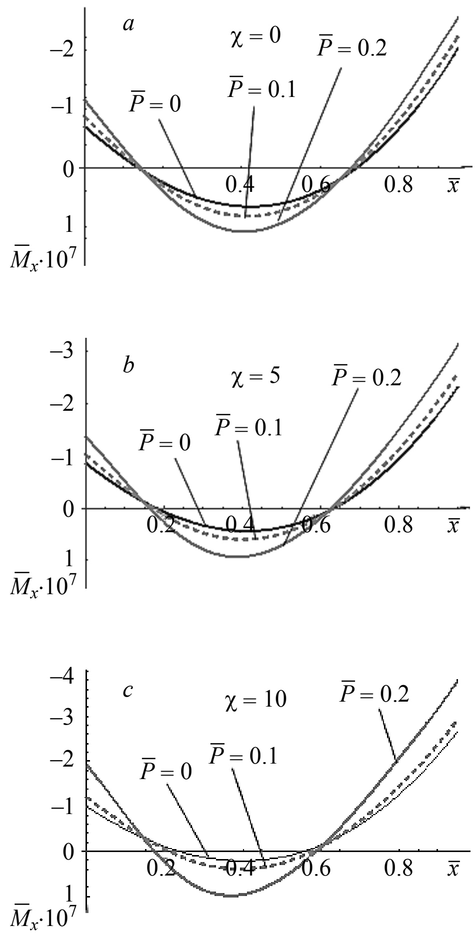

Fig. 5. Distribution of the bending moment: $\chi =$ (*a*) 0, (*b*) 5, and (*c*) 10

Рис. 5. Распределение изгибающего момента: $a - \chi = 0$; $b - \chi = 5$; $c - \chi = 10$

Conclusions

A mathematical model has been developed to solve the problem of bending and stability of an elastically clamped beam. The solution to the resulting system of differential equations is based on the collocation method. Unknown functions are approximated by polynomials. In numerical calculations, the stability of solutions is studied depending on the degree of polynomials.

It is found that the maximum point of bending of the beam is located on its thin side. An increase in compressive force leads to an increase in bending. Consideration of transverse shear does not significantly affect the varying behavior of the transverse force and bending moment.

The results are expected to be useful for engineers and builders.

REFERENCES

1. Volmir. A.S. (1967) *Ustoychivost' deformiruemykh sistem* [Stability of deformable systems]. Moscow: Nauka.
2. Kirakosyan R.M., Stepanyan S.P. (2014) Neklassicheskaya zadacha izgiba ortotropnoy balki peremennoy tolshchiny s uprugo-zashchemlennoy oporoy [Non-classical problem of bending an orthotropic beam of variable thickness with an elastically clamped support]. *Natsional'naya Akademiya Nauk Armenii. Doklady – National Academy of Sciences of Armenia*. *Reports.* 114(3). pp. 205–212.
3. Kirakosyan R.M. (2000) *Prikladnaya teoriya ortotropnykh plastin peremennoy tol'shchiny, uchityvayushchaya vliyanie deformatsiy poperechnykh sdvigov* [Applied theory of orthotropic plates of variable thickness with allowance for the influence of transverse shear strains]. Yerevan: Nauka.
4. Ambartsumyan S.A. (1987) *Teoriya anizotropnykh plastin* [Theory of anisotropic plates]. Moscow: Nauka.
5. Gomez H., de Lorenzis L. (2016) The variational collocation method. *Computer Methods in Applied Mechanics and Engineering*. 309. pp. 152–181. DOI: 10.1016/j.cma.2016.06.003.
6. Kirakosyan R.M. (2015) *Ob odnoy neklassicheskoy zadache izgiba uprugo zashchemlennoy krugloy plastinki* [On one non-classical problem of a bend of an elastically clamped round plate]. *Natsional'naya Akademiya Nauk Armenii. Doklady – National Academy of Sciences of Armenia*. *Reports.* 115(4). pp. 284–289.
7. Kirakosyan R.M., Stepanyan S.P. (2016) *Neklassicheskaya kraevaya zadacha uprugo zashchemlionnoy krugloy ortotropnoy plastinki* [Non-classical boundary value problem of an elastically clamped partially loaded round orthotropic plate]. *Izvestiya Natsional'noy Akademii Nauk Armenii. Mekhanika – Proceedings of National Academy of Sciences of Armenia. Mechanics.* 69(3). pp. 59–70. DOI: 10.33018/69.3.12.
8. Kirakosyan R.M., Stepanyan S.P. (2017) *Ustoychivost' sterzhnya pri uchiote umen'sheniya szhimayushchey sily uprugo zashchemlionnoy oporoy* [Stability of the rod with allowance for the reduction of the compressive force of an elastically clamped support]. *Izvestiya Natsional'noy Akademii Nauk Armenii. Mekhanika – Proceedings of National Academy of Sciences of Armenia. Mechanics.* 70(3). pp. 57–66. DOI: 10.33018/70.3.5.
9. Kirakosyan R.M., Stepanyan S.P. (2017) Non-classical problem of bend of an orthotropic annular plate of variable thickness with an elastically clamped support. *Proceedings of the Yerevan State University*. *Physical and Mathematical Sciences*. 51(2). pp. 168–176. DOI: 10.46991/PYSU:A/2017.51.2.168.
10. Kirakosyan R.M., Stepanyan S.P. (2018) The non-classical problem of an elastically clamped orthotropic beam of compressive forces and transverse load. *Proceedings of the Yerevan State University*. *Physical and Mathematical Sciences*. 52(2). pp. 101–108. DOI: 10.46991/PYSU:A/2018.52.2.101.
11. Kirakosyan R.M., Stepanyan S.P. (2019) The non-classical problem of an orthotropic beam of variable thickness with the simultaneous action of its own weight and compressive axial forces. *Proceedings of the Yerevan State University*. *Physical and Mathematical Sciences*. 53(3). pp. 183–190. DOI: 10.46991/PYSU:A/2019.53.3.183.
12. Batista M. (2019) Stability of clamped-elastically supported elastic beam subject to axial compression. *International Journal of Mechanical Sciences*. 155. pp. 1–8. DOI: 10.1016/j.ijmecsci.2019.02.030.
13. Wagner H.N.R., Sosa E.M., Ludwig T., Croll J.G.A, Hühne C. (2019) Robust design of imperfection sensitive thin-walled shells under axial compression, bending or external pressure. *International Journal of Mechanical Sciences*. 156. pp. 205–220. DOI: 10.1016/j.ijmecsci.2019.02.047.
14. Dang X., He K., Zhang F., Zuo Q., Du R. (2019) Multi-stage incremental bending to form doubly curved metal plates based on bending limit diagram. *International Journal of Mechanical Sciences*. 155. pp. 19–30. DOI: 10.1016/j.ijmecsci.2019.02.001.

15. Song Z., Cao Q., Dai Q. (2019) Free vibration of truncated conical shells with elastic boundary constraints and added mass. *International Journal of Mechanical Sciences*. 155. pp. 286–294. DOI: 10.1016/j.ijmecsci.2019.02.039.
16. Liu Z., Yang C., Gao W., Wu D., Li G. (2019) Nonlinear behaviour and stability of functionally graded porous arches with graphene platelets reinforcements. *International Journal of Engineering Science*. 137. pp. 37–56. DOI: 10.1016/j.ijengsci.2018.12.003.
17. Berchio E., Buoso D., Gazzolla F., Zucco D. (2018) A minimaxmax problem for improving the torsional stability of rectangular plates. *Journal of Optimization Theory and Applications*. 177(1). pp. 64–92. DOI: 10.1007/s.10957-018-1261-1.
18. Stephan E.P., Teltscher M.T. (2018) Collocation with trigonometric polynomials for integral equations to the mixed boundary value problem. *Numerische Mathematik*. 140(1). pp. 153–190. DOI: 10.1007/s00211-018-0960-8.
19. Dorogov Yu.I. (2016) Ustoychivost' gorizontal'nogo uprugogo sterzhnya [StabIility of a horizontal elastic bar]. *Vestnik Tomskogo gosudarstvennogo universiteta. Matematika i mekhanika – Tomsk State University Journal of Mathematics and Mechanics*. 4(42). pp. 70–83. DOI: 10.17223/19988621/42/7.
20. Ishaquddin Md., Gopalakrishnan S. (2019) A novel weak form quadrature element for gradient elastic beam theories applied. *Mathematical Modelling*. 77(1). pp. 1–16. DOI: 10.1016/j.apm.2019.07.014.
21. Song H., Yang Z., Brunner H. (2019) Analysis of collocation methods for nonlinear Volterra integral equations of the third kind. *Calcolo*. 56(1). pp. 1–29. DOI: 10.1007/s10092-019-0304-9.
22. Papargyri-Beskou S., Tsepoura K.G., Polyzos D., Beskos D.E. (2003) Bending and stability analysis of gradient elastic beams. *International Journal of Solids and Structures*. 40(2). pp. 385–400. DOI: 10.1016/S0020-7683(02)00522-X.
23. Shcherbakov I.V., Lyukshin B.A. (2019) Modelirovanie povedeniya otklika ortotropnoy plastiny pri vozdeystvii dinamicheskoy nagruzki [Simulation of the behavior of an orthotropic plate response under dynamic load]. *Vestnik Tomskogo gosudarstvennogo universiteta. Matematika i mekhanika – Tomsk State University Journal of Mathematics and Mechanics*. 61. pp. 111–118. DOI: 10.17223/19988621/61/10.
24. Grygorowicz M., Magnucka-Blandzi E. (2016) Mathematical modeling for dynamic stability of sandwich beam with variable mechanical properties of core. *Applied Mathematics and Mechanics*. 37(10). pp. 1361–1374. DOI: 10.1007/s10483-016-2137-9.
25. Chen Y., Yao W. (2016) Mechanical model of round window membrane under reverse excitation. *Applied Mathematics and Mechanics*. 37(10). pp. 1341–1348. DOI: 10.1007/s10483-016-2136-9.
26. Musa A.E.S. (2017) Galerkin method for bending analysis of beams on non-homogeneous foundation. *Journal of Applied Mathematics and Computational Mechanics*. 16(3). pp. 61–72. DOI: 10.17512/jamcm.2017.3.06.
27. Sowa L., Pawel K. (2017) Mathematical modeling of mechanical phenomena in the gantry crane beam. *Journal of Applied Mathematics and Computational Mechanics*. 16(3). pp. 97–104. DOI: 10.17512/jamcm.2017.3.09.
28. Lazarev N.P., Rudoy E.M. (2017) Optimal size of a rigid thin stiffener reinforcing an elastic plate on the outer edge. *Journal of Applied Mathematics and Mechanics*. 97(9). pp. 1120–1127. DOI: 10.1002/zamm.201600291.

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Ключевые слова: упруго-защемленная опора, изгиб, поперечный сдвиг, устойчивость.

На основе уточненной теории ортотропных пластин переменной толщины, построена математическая модель задачи изгиба и устойчивости упруго защемленной балки. Для решения задачи в случае одновременного действия собственного веса и сжимающих осевых сил получена система дифференциальных уравнений с переменными коэффициентами. Учитываются также влияния поперечного сдвига и уменьшения сжимающей силы опоры. Переходя к безразмерным величинам, методом коллокаций решается конкретная задача для балки линейно изменяющейся толщины. Неизвестные функции аппроксимируются полиномами. В численных расчетах исследуется устойчивость решений в зависимости от степени полиномов. Обсуждается устойчивость балки, величина критической силы определяется изменением значения осевой сжимающей силы до тех пор, пока величина прогиба не изменит знак. Результаты представлены как в табличной, так и в графической формах. По полученным результатам сделаны соответствующие выводы. В частности выяснилось, что: а) максимальная точка изгиба балки находится на ее тонкой стороне. Увеличение сжимающей силы приводит к увеличению прогиба; б) учет поперечного сдвига не оказывает значительного влияния на изменение поведения поперечной силы и изгибающего момента. Полученные результаты будут полезны инженерам и строителям.

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