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Optimal design of reliability experiment based on the Wiener degradation model with covariates

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Abstract. The Wiener degradation models with covariates are widely used for reliability analysis. In this paper, an algorithm for constructing an optimal design, which includes determining optimal stress levels, number of tested devices and time moments for measuring the degradation index, has been developed. The proposed algorithm is based on the optimization of some functional of the Fisher information matrix under restrictions on stress levels, experiment duration and minimum time interval between measurements of the degradation index. Moreover, the example of LED degradation analysis has been considered.
Keywords: Wiener degradation model; covariates; Fisher information matrix; optimal design; light-emitting diodes

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Оптимальное планирование эксперимента на основе винеровской деградационной модели с ковариатами

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Аннотация. Винеровские деградационные модели с ковариатами широко используются для оценки функции надежности по данным об изменении показателя деградации во времени. В данной статье разработан алгоритм построения оптимального плана эксперимента на надежность, который предусматривает вычисление оптимальных величин нагрузок, количества исследуемых изделий и моментов времени измерения показателя деградации. Предложенный алгоритм основан на оптимизации функционала от информационной матрицы Фишера при заданных ограничениях на величину нагрузки, длительность проведения эксперимента и минимальный интервал между моментами времени измерения показателя деградации. Применение разработанного алгоритма рассмотрено на примере данных об исследовании светодиодов (LED).

Ключевые слова: винеровская деградационная модель; ковариаты; информационная матрица Фишера; оптимальное планирование эксперимента; светодиоды

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Among all degradation models proposed in literature (see, for example, [1]), the most popular degradation models are the Wiener, gamma and inverse Gaussian models [2, 3]. Gamma [4–6] and inverse Gaussian [7, 8] degradation models are widely used for describing the aging processes of equipment, if the degradation index changes monotonously. On the contrary, the Wiener degradation model can be used in the case of non-monotonic degradation, when there are both positive and negative increments of the degradation index. The Wiener degradation models are widely used in various applications [9–14]. For example, in [12] it is applied for the reliability analysis of hard disk head units, in [13, 14] it is used to describe the degradation of LED.

The modern world requires development of new high-tech devices during extremely restricted period of time. At the same time, productivity, reliability and overall quality must be constantly improved. It has encouraged the wider use of optimal design to improve the quality of devices and processes in general. High reliability requirements have increased the need for testing materials, components and systems in early stages.

Evaluating the lifetime distribution and the reliability characteristics of components in high-tech devices is an essentially challenging task. Most modern devices are designed for operation without failure during long period of time. Thus, under normal conditions, most objects will maintain functionality. For example, in the design and construction of a communication satellite, there are only 6 months to test components which are expected to operate for 15 – 20 years. For this reason, accelerated testing is widely used for industrial purposes, in particular for obtaining well-timed information on the reliability of components and product materials. Usually, results obtained from testing under high levels of stresses (such as temperature, voltage, pressure and others) are extrapolated using a physically based statistical model to provide estimates of reliability characteristics under normal conditions of use.

The issue of constructing an optimal design has been raised by many scientists since the middle of the 20th century. Over time, scientists have noticed that traditional research methods are ineffective and costly. Therefore, scientists began to look for new ways to speed up testing and ensure that made decisions are close to optimal. The English statistician Sir Ronald Aylmer Fisher introduced fresh ideas into the planning experiments [15]. He was the first to show the expediency of simultaneous variation by all factors as opposed to the widespread ‘vary one factor at a time’ approach with other factors assigned fixed values. Since then, a new era of optimal design has begun, which is still relevant in our time. At the beginning of the 21st century, scientists discussed optimal design of degradation tests in the presence of cost constraint. In [16], the authors proposed an approach to determine the number of units to test and stress levels by minimizing the variance of estimated percentile of failure time distribution under determined cost of experiment and degradation test duration. An example of the optimal design for the accelerated reliability experiment on the basis of the Wiener degradation model was considered in [17]. However, there are no recommendations for the time moments for measuring the degradation index.

In [18], we showed that the choice of time moments for measuring the degradation index significantly influence on the accuracy of maximum likelihood estimates of the Wiener degradation model parameters. The optimal distribution of measurement time points depends on the model describing the degradation process as well as the experimental conditions, such as stress levels, experiment duration and minimum time interval between measurements of the degradation index [18–20]. Thus, the main purpose of the paper is to develop an algorithm for constructing A- and D-optimal designs based on the Wiener degradation model, which includes determining of optimal stress levels, number of tested devices and time moments for measuring the degradation index.

1. The Wiener degradation model in reliability analysis

Let us assume that the observed stochastic process $Z(t)$ is a stochastic process with independent increments and $Z(0)=0$. For the Wiener degradation model, increments have the normal distribution with the probability density function

$$f(u,\theta_1,\theta_2)=\frac{1}{\theta_2\sqrt{2\pi}}\exp\left(-\frac{(u-\theta_1)^2}{2\theta_2^2}\right),$$

where $\theta_1=\mu\big(\rho(t+\Delta t)-\rho(t)\big)$ is the shift parameter, $\theta_2=\sigma\sqrt{\rho(t+\Delta t)-\rho(t)}$ is the scale parameter, $\sigma>0$, $\rho(t)$ is a positive increasing function.

Let us denote the vector of stresses (which are also often referred to as covariates) as $x=\left(x^1,x^2,...,x^m\right)^{\mathrm{T}}$. The range of values for each covariate x^j, $j=\overline{1,m}$ is determined by the conditions of experiment. In this paper, the degradation process $Z(t)$ is supposed to be observed under a constant in time stress.

Here, we assume that the covariate *x* influences the degradation paths as in the accelerated failure time model [5]:

$$Z_x(t)=Z\left(\frac{t}{r(x;\beta)}\right),$$

where $r(x;\beta)$ is a the positive covariate function, $\beta=(\beta_1,...,\beta_m)^{\mathrm{T}}$ is the vector of regression parameters.

Denote the mathematical expectation of degradation process $Z_x(t)$ by

$$\mathrm{E}\big(Z_x(t)\big)=\mu\rho\left(\frac{t}{r(x;\beta)},\gamma\right).$$

The time to failure, which depends on covariate x is defined as:

$$\tau=\sup\left\{t:Z_x(t)<z_0\right\},$$

where z_0 is the critical value of the degradation index. Then, the reliability function can be represented as:

$$S(t)=P\{\tau>t\}=P\left\{Z_x(t)<z_0\right\}=\Phi\left(\frac{z_0-\mu\rho\big(t/r(x;\beta);\gamma\big)}{\sigma\sqrt{\rho\big(t/r(x;\beta);\gamma\big)}}\right).$$

Suppose the experiment is running over time T. The degradation index values are measured at time points $0=t_0<t_1...<t_k=T$.

Let us denote the sample of independent degradation index increments with covariates as following:

$$\mathbf{X}_n=\left\{\left(\Delta Z_{1j},x_1\right),\left(\Delta Z_{2j},x_2\right),...,\left(\Delta Z_{nj},x_n\right),\,j=\overline{1,k}\right\},$$

where *k* is the number of measurements of the degradation index for each object, x_i is the value of the covariate vector for the *i*-th object, $\Delta Z_{ij}=Z_i\left(t_j\right)-Z_i\left(t_{j-1}\right)$ is the increment of the degradation index during the time from t_{j-1} to t_j.

Unknown parameters of the model can be estimated using the maximum likelihood method. The logarithmic likelihood function for the parameters of the Wiener degradation model is defined as:

$$\ln L(\mathbf{X}_n)=-nk\left(\ln\sqrt{2\pi}+\ln\sigma\right)-\frac{n}{2}\sum_{j=1}^{k}\ln\Big(\rho\big(t_{j+1}\big)-\rho\big(t_j\big)\Big)-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\sum_{j=1}^{k}\frac{\Big[\Delta Z_{ij}-\mu\Big(\rho\big(t_{j+1}\big)-\rho\big(t_j\big)\Big)\Big]^2}{\Big(\rho\big(t_{j+1}\big)-\rho\big(t_{j+1}\big)\Big)}. \quad (1)$$

The maximum likelihood estimate (MLE) of an unknown parameter corresponds to maximum of the likelihood function (1). For solving the optimization problem, we have calculated the derivatives of the log-likelihood function with respect to the model parameters:

$$\begin{cases} \dfrac{\partial \ln L}{\partial \sigma} = -\dfrac{nk}{\sigma} + \dfrac{1}{\sigma^3} \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{k} \dfrac{\left(\Delta Z_{ij} - \mu\left(\Delta\rho\left(t_j\right)\right)\right)^2}{\left(\Delta\rho\left(t_j\right)\right)} = 0; \\ \dfrac{\partial \ln L}{\partial \mu} = \dfrac{1}{\sigma^2}\left(\mu n \sum\limits_{j=1}^{k} \Delta\rho\left(t_j\right) - \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{k} \Delta Z_{ij}\right) = 0; \\ \dfrac{\partial \ln L}{\partial \gamma} = -\dfrac{n}{2} \sum\limits_{j=1}^{k} \dfrac{\left(\dfrac{\partial \Delta\rho\left(t_j\right)}{\partial \gamma}\right)}{\left(\Delta\rho\left(t_j\right)\right)} - \dfrac{1}{2\sigma^2} \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{k} \left(\mu^2 \left(\dfrac{\partial \Delta\rho\left(t_j\right)}{\partial \gamma}\right) - \dfrac{\Delta Z_{ij}^2 \left(\dfrac{\partial \Delta\rho\left(t_j\right)}{\partial \gamma}\right)}{\left(\Delta\rho\left(t_j\right)\right)^2}\right) = 0; \\ \dfrac{\partial \ln L}{\partial \beta} = -\dfrac{n}{2} \sum\limits_{j=1}^{k} \dfrac{\left(\dfrac{\partial \Delta\rho\left(t_j\right)}{\partial \beta}\right)}{\left(\Delta\rho\left(t_j\right)\right)} - \dfrac{1}{2\sigma^2} \sum\limits_{i=1}^{n} \sum\limits_{j=1}^{k} \left(\mu^2 \left(\dfrac{\partial \Delta\rho\left(t_j\right)}{\partial \beta}\right) - \dfrac{\Delta Z_{ij}^2 \left(\dfrac{\partial \Delta\rho\left(t_j\right)}{\partial \beta}\right)}{\left(\Delta\rho\left(t_j\right)\right)^2}\right) = 0. \end{cases} \quad (2)$$

In the general case, system of equations (2) is solved using numerical methods. In the case of a model without covariates with the linear trend function, i.e. $\gamma = 1$, $\beta = 0$, the MLE of parameters have the following form:

$$\hat{\mu} = \frac{\sum\limits_{i=1}^{n} \sum\limits_{j=1}^{k} \Delta Z_{ij}}{nT}, \hat{\sigma} = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{k} \frac{(\Delta Z_{ij} - \hat{\mu}(t_{j+1} - t_j))^2}{\left(t_{j+1} - t_j\right)}}.$$

2. Optimal design of experiments

As shown in [18], the accuracy of the estimates of unknown parameters significantly depends on the conditions of reliability experiment - stress levels, the number of tested devices and the time moments for measuring the degradation index. Therefore, the stage of experiment design is very important in practice.

2.1. The problem of optimal design for reliability experiments

We denote the experiment design as a set of values:

$$\xi = \left\{ \begin{matrix} x_{(1)} & \dots & x_{(q)} \\ \omega_1 & \dots & \omega_q \end{matrix}, t_1 \dots t_k \right\},$$

where $x_{(1)}, \dots, x_{(q)}$ are the reference points of the plan, which are necessarily different; $\omega_i = \dfrac{n_i}{n}$, $\omega_i \ge 0$, $\sum\limits_{i=1}^{q} \omega_i = 1$, where n_i is the number of objects (individuals) examined under the stress level $x_{(i)}$, $i = \overline{1, q}$, $q \le n$, $\sum\limits_{i=1}^{q} n_i = n$, and $t_1 \dots t_k$ are the time moments for measuring the degradation index. All objects of the sample are divided into q groups corresponding to different values of the covariate vector (reference points of the design).

Thus, the problem of an optimal design can be written as follows:

$$\begin{cases} M\left(I(\xi)\right) \to \min, \\ x_{\min} \le x_{(i)} \le x_{\max}, \\ t_0 = 0, t_k = T, t_j - t_{j-1} > \Delta t_{\min}, j = \overline{1,k}, \end{cases}$$

where $M(\cdot)$ is some functional of the Fisher information matrix, $x_{\min}$, $x_{\max}$ are the minimum and maximum values of stress levels determined by the conditions of experiment, $\Delta t_{\min}$ is the minimum interval between adjacent time points of measuring the degradation index.

There are various optimal designs: A, D, G, Q, etc. In this paper, we consider the problem of experiment design from the standpoint of increasing the accuracy of the model parameters estimates. The procedures for constructing A- and D- optimal designs make it possible to obtain a redistribution of suitable candidate points taking into account the extraction of the maximum information about the model parameters from the experimental data.

The construction of the A-optimal design consists in minimizing the sum of the diagonal elements of the inverse Fisher matrix:

$$M\left(I(\xi)\right) = \mathrm{Trace}\left(I^{-1}(\xi)\right).$$

The A-optimal design corresponds to the scattering ellipsoid of the parameter estimates with the least sum of squares of the axes lengths.

The construction of the D-optimal design is based on maximizing the determinant of the Fisher information matrix:

$$M\left(I(\xi)\right) = -\det\left(I(\xi)\right).$$

The scattering ellipsoid of the parameter estimates corresponding to the D-optimal design has the minimum volume.

2.2. Fisher information matrix for the Winner degradation model

To obtain the Fisher information matrix for the Wiener degradation model, it is necessary to calculate the mathematical expectation of the second derivatives with respect to the parameters of the likelihood function. The first-order derivatives of the likelihood function are represented by formula (2).

Elements of the Fisher information matrix are given by the following formulas:

$$I_{11} = \frac{2}{\sigma^2} nk;$$

$$I_{12} = I_{21} = 0;$$

$$I_{13} = I_{31} = \frac{1}{\sigma} \sum_{i=1}^{q} \sum_{j=1}^{k} \frac{1}{\Delta\rho_{ij}} \cdot \frac{\partial \Delta\rho_{ij}}{\partial \gamma};$$

$$I_{14} = I_{41} = \frac{1}{\sigma} \sum_{i=1}^{q} \sum_{j=1}^{k} \frac{1}{\Delta\rho_{ij}} \cdot \frac{\partial \Delta\rho_{ij}}{\partial \beta};$$

$$I_{22} = \frac{1}{\sigma^2} \sum_{i=1}^{q} \sum_{j=1}^{k} \Delta\rho_{ij};$$

$$I_{23} = I_{32} = \frac{\mu^2}{\sigma^2} \sum_{i=1}^{q} \sum_{j=1}^{k} \frac{\partial \Delta\rho_{ij}}{\partial \gamma};$$

$$I_{24} = I_{42} = \frac{\mu^2}{\sigma^2} \sum_{i=1}^{q} \sum_{j=1}^{k} \frac{\partial \Delta\rho_{ij}}{\partial \beta};$$

$$I_{33} = \sum_{i=1}^{q} \sum_{j=1}^{k} \left(\frac{\partial \Delta\rho_{ij}}{\partial \gamma}\right)^2 \cdot \left(\frac{1}{2\Delta\rho_{ij}^2} + \frac{\mu^2}{\sigma^2} \cdot \frac{1}{\Delta\rho_{ij}}\right);$$

$$I_{34} = I_{43} = \sum_{i=1}^{q} \sum_{j=1}^{k} \left(\frac{\partial \Delta\rho_{ij}}{\partial \gamma}\right) \cdot \left(\frac{\partial \Delta\rho_{ij}}{\partial \beta}\right) \cdot \left(\frac{1}{2\Delta\rho_{ij}^2} + \frac{\mu^2}{\sigma^2} \cdot \frac{1}{\Delta\rho_{ij}}\right);$$

$$I_{44} = \sum_{i=1}^{q} \sum_{j=1}^{k} \left(\frac{\partial \Delta\rho_{ij}}{\partial \beta}\right)^2 \cdot \left(\frac{1}{2\Delta\rho_{ij}^2} + \frac{\mu^2}{\sigma^2} \cdot \frac{1}{\Delta\rho_{ij}}\right);$$

where

$$\Delta\rho_{ij} = \rho\left(\frac{t_{ij+1}}{r(x_i;\beta)},\gamma\right) - \rho\left(\frac{t_{ij}}{r(x_i;\beta)},\gamma\right).$$

2.3. Algorithm of the direct search procedure for optimal design

The direct approach assumes solving the optimization problem:

$$\xi^* = \arg\min M\left(I(\xi)\right).$$

Step 1. Set the initial non-degenerate design:

$$\xi^0 = \left\{ \begin{matrix} x_{(1)}^0 & \dots & x_{(q)}^0 \\ \omega_1^0 & \dots & \omega_q^0 \end{matrix}, t_1^0 \dots t_k^0 \right\}.$$

Set $iter = 0$.

Step 2. Calculate the Fisher information matrix $I\left(\xi^0\right)$ for the initial design.

Step 3. Fix the values of $\omega_1^0,\dots,\omega_q^0$, $t_1^0,\dots,t_k^0$, and solve the optimization problem

$$M\left(I(\xi)\right) \to \min_{x_{(1)}\dots x_{(q)}.} .$$

Calculate $I\left(\xi^{iter}\right)$ according to the received design:

$$\xi^{iter} = \left\{ \begin{matrix} x_{(1)}^{iter+1} & \dots & x_{(q)}^{iter+1} \\ \omega_1^{iter} & \dots & \omega_q^{iter} \end{matrix}, t_1^{iter} \dots t_k^{iter} \right\}.$$

Step 4. Fix the values of $x_{(1)}^{iter+1},\dots,x_{(q)}^{iter+1}$, $t_1^{iter},\dots,t_k^{iter}$, and solve the optimization problem

$$M\left(I(\xi)\right) \to \min_{\omega_1,\dots,\omega_q} .$$

Calculate $I\left(\xi^{iter}\right)$ according to the received design:

$$\xi^{iter} = \left\{ \begin{matrix} x_{(1)}^{iter+1} & \dots & x_{(q)}^{iter+1} \\ \omega_1^{iter+1} & \dots & \omega_q^{iter+1} \end{matrix}, t_1^{iter} \dots t_k^{iter} \right\}.$$

Step 5. Fix the values of $x_{(1)}^{iter+1},\dots,x_{(q)}^{iter+1}$, $\omega_1^{iter+1},\dots,\omega_q^{iter+1}$, and solve the optimization problem:

$$M\left(I(\xi)\right) \to \min_{t_1,\dots,t_k} ,$$

Calculate $I\left(\xi^{iter}\right)$ according to the received design.

Step 6. Check the termination condition for the obtained design

$$\xi^{iter+1} = \left\{ \begin{matrix} x_{(1)}^{iter+1} & \dots & x_{(q)}^{iter+1} \\ \omega_1^{iter+1} & \dots & \omega_q^{iter+1} \end{matrix}, t_1^{iter+1} \dots t_k^{iter+1} \right\}.$$

If for a small positive number ε the inequality:

$$\left| M\left(\xi^{iter+1}\right) - M\left(\xi^{iter}\right) \right| \le \varepsilon$$

holds, then the optimal design is obtained; else set $iter = iter + 1$ and repeat steps 3-6.

3. Optimal design for the light-emitting diodes reliability test

Light-emitting diodes have higher brightness and lower power consumption than traditional light sources. In [21], the degradation of light-emitting diodes was studied under two electric current levels: 35 mA and 40 mA, while the normal stress level is 25 mA. Light intensity data for 24 light-emitting diodes are shown in Table 1. As can be seen from the table, the output light intensity decreases with time. The data were recorded every 50 hours up to 250 hours. When the light intensity decreases by 50 percent the failure of unit is recorded.

Table 1

Degradation data of LED

№	I	$t_0 = 0$	$t_1 = 50$	$t_2 = 100$	$t_3 = 150$	$t_4 = 200$	$t_5 = 250$
1	40	100	86,6	78,7	76,0	71,6	68,0
2		100	82,1	71,4	65,4	61,7	58,0
3		100	82,7	70,3	64,0	61,3	59,3
4		100	79,8	68,3	62,3	60,0	59,0
5		100	75,1	66,7	62,8	59,0	54,0
6		100	83,7	74,0	67,4	63,0	61,3
7		100	73,0	65,0	60,7	58,3	58,0
8		100	86,2	67,6	62,7	60,0	59,7
9		100	81,2	65,0	60,6	59,3	57,3
10		100	66,1	64,2	59,4	58,0	55,3
11		100	76,5	61,7	61,3	59,7	56,0
12		100	66,8	63,3	59,3	57,3	56,5
13	35	100	95,1	86,0	77,6	70,0	66,7
14		100	93,3	87,1	79,7	74,3	73,0
15		100	98,3	92,4	89,0	84,3	83,0
16		100	96,6	88,2	85,1	81,4	78,6
17		100	95,8	89,0	84,0	81,0	80,0
18		100	94,0	82,4	77,4	71,7	70,6
19		100	88,2	78,7	75,0	70,0	69,3
20		100	86,7	78,0	73,3	68,7	67,3
21		100	89,0	80,0	76,3	72,3	71,3
22		100	96,2	86,5	81,4	74,5	74,2
23		100	97,5	84,5	81,0	75,0	74,1
24		100	92,4	85,4	80,0	73,3	71,5

The degradation model describing the reliability function can be constructed by the given preliminary data. Then, on the basis of the obtained model it is proposed to determine the optimal design, which enables to increase the precision of model parameters estimates.

Let us consider the Wiener degradation model with the power-law trend function $\rho(t) = t^{\gamma}$ for different covariate functions:

1. Log-linear model: $r_1(x, \beta) = \exp(\beta \cdot x)$;
2. Power model: $r_2(x, \beta) = x^{\beta} = \exp(\beta \cdot \ln(x))$, $x > 0$;
3. Arrhenius model: $r_3(x, \beta) = \exp(\beta / x)$.

As the Wiener degradation model described above requires $Z(0) = 0$ and the increasing trend function, the data were processed as follows: the values of degradation index are equal to 100 minus values of light intensity, given in Table 1.

Maximum likelihood estimates of unknown parameters for the considered Wiener degradation models as well as obtained values of information criteria AIC and BIC are given in Table 2.

Table 2

Estimation Results of Unknown Parameters $(\sigma, \mu, \gamma, \beta)$ for the Wiener degradation model with the power-law trend

Covariate function	Log-linear	Power	Arrhenius
Parameters estimates	[0,52; 0,13; 0,48; –6,46]	[0,84; 0,33; 0,48; –1,72]	[18,00; 156,45; 0,48; 24,16]
AIC	646	648	644
BIC	658	660	656

As can be seen from Table 2, the more preferable model is the Wiener degradation model with power-law trend function, where the influence of the current strength is described by log-linear covariate function. Figures 1 and 2 illustrate the graphs of trend functions and the values of degradation index under stress levels 35 and 40 mA.

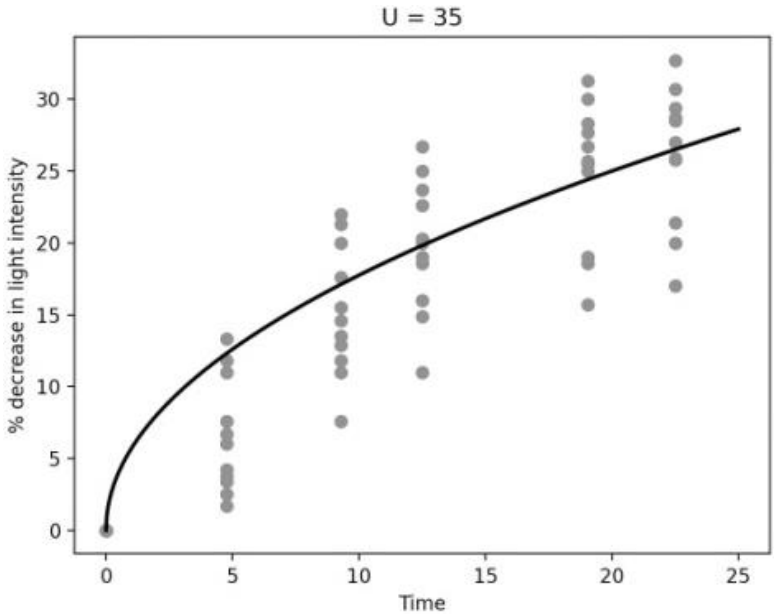

Fig. 1. Degradation processes at a current of 35

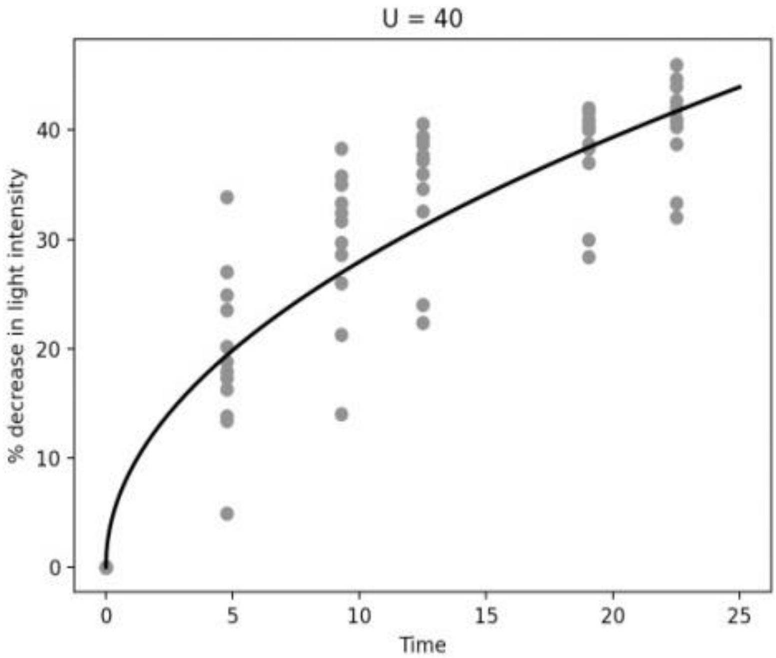

Fig. 2. Degradation processes at a current of 40

On the basis of the obtained Wiener degradation model we build the optimal design for the further experiment. Let us find the optimal experiment design using the proposed algorithm presented in Section 2.3 as a result of solving the optimization problem

$$\begin{cases} M\left(I\left(\xi\right)\right) = -\det\left(I\left(\xi\right)\right) \to \min, \\ 30 \le x_{(i)} \le 50,\ i = \overline{1,q}, \\ t_0 = 0,\ t_k \le 400,\ t_{j+1} - t_j > 10,\ j = \overline{0,k}, \end{cases}$$

where $q = 2$ and $k = 5$.

In Table 3, there are the initial design and obtained D-optimal design as well as the corresponding values of determinant of the Fisher information matrix.

Table 3

D-Optimal Experiment Design

Initial design	Optimal design
$\xi^0 = \left\{ \begin{matrix} 35 & 40 \\ 0{,}5 & 0{,}5 \end{matrix},\ 0\ \ 50\ \ 100\ \ 150\ \ 200\ \ 250 \right\}$	$\xi^* = \left\{ \begin{matrix} 30 & 50 \\ 0{,}5 & 0{,}5 \end{matrix},\ 0\ \ 10\ \ 44\ \ 150\ \ 360\ \ 500 \right\}$
$\det\left(I\left(\xi^0\right)\right) = 6{,}7e+09$	$\det\left(I\left(\xi^*\right)\right) = 1e+12$

The purpose of developing the proposed algorithm is to select the optimal experimental conditions in order to improve the accuracy of model parameters estimates. Let us analyze statistical properties of model parameters estimates using the Monte Carlo method. We generated $N = 100000$ samples of degradation paths following the initial and optimal designs. In Table 4, there are the values of determinant of the estimated covariance matrix

$$\left\| \frac{1}{N} \sum_{l=1}^{N} \left(\hat{\theta}_l^i - \bar{\hat{\theta}}^i\right)\left(\hat{\theta}_l^j - \bar{\hat{\theta}}^j\right) \right\|_{4\times 4},$$

where $\left(\theta^1, \theta^2, \theta^3, \theta^4\right) = \left(\sigma, \mu, \gamma, \beta\right)$, $\hat{\theta}_l^i$ is the maximum likelihood estimate of parameter θ^i obtained from the *l*-th sample and $\bar{\hat{\theta}}^i = \frac{1}{N} \sum_{l=1}^{N} \hat{\theta}_l^i$.

Table 4

Determinant of the estimated covariance matrix

Initial design	Optimal design
3,35e-08	1,75e-12

As can be seen from Table 4, the determinant of the estimated covariance matrix has decreased tenfold for the optimal design, which confirms that the parameters estimates for the optimal design are indeed more accurate.

Conclusion

In this paper, we have proposed the algorithm for direct search of optimal design basing on the Wiener degradation model. The algorithm enables to determine the optimal stress levels, number of tested devices and time moments for measuring the degradation index. As an example, we have considered the problem of reliability analysis for light-emitting diodes. Following the proposed algorithm, the D-optimal design for testing reliability of light-emitting diodes has been obtained. It has been shown that the determinant of the estimated covariance matrix has decreased tenfold for the optimal design in comparison with the initial design, which confirms accuracy of parameter estimates has become higher.

However, in this paper we have not taken into account the fact that the object observation should be terminated when the degradation path reaches the threshold. Our further research is associated with the improvement of the algorithm on the basis of the conditional Fisher information matrix, which can result in the change of the optimal design.

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