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Observer-based adaptive output tracking for MIMO LTI systems under unknown external disturbances

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Abstract. The paper presents an adaptive output-feedback control algorithm for linear time-invariant multi-input multi-output systems subjected to unmatched unknown disturbances. The control algorithm is based on the Falb–Wolovich decoupling method combined with a direct disturbance compensation technique derived from the internal model principle. The main advantage of the proposed algorithm is that the adaptive law becomes easier to tune due to the decoupling strategy application. By properly selecting the adaptation gains independently the balance between the convergence speed and robustness according to the desired performance requirements can be achieved. In addition, it is possible to get improved control performance in situations where negative inter-channel coupling of an original system has an adverse effect to the overall dynamic of the system. The effectiveness of the proposed method is demonstrated through numerical simulations performed in MATLAB/Simulink.

Keywords: adaptive control; disturbance compensation; Falb-Wolovich approach; harmonic disturbance; Luenberger observer.

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Адаптивное слежение выходной переменной за задающим сигналом в многоканальных системах с наблюдаемым выходом в условиях неизвестных внешних возмущений

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Аннотация. Представлен алгоритм адаптивного слежения выходной переменной за задающим сигналом в многоканальных системах с наблюдаемым выходом в условиях несогласованных неизвестных возмущений. Алгоритм основан на использовании метода развязки Фальба–Воловича в сочетании с прямым методом компенсации возмущений на основе принципа внутренней модели. Основное преимущество предлагаемого алгоритма заключается в том, что он более прост в настройке благодаря «развязанной» структуре каналов управления. Независимый выбор коэффициентов адаптации для каждого из каналов системы позволяет обеспечить требуемые показатели по скорости сходимости и свойство робастности системы. Кроме того, рассматриваемый в статье подход позволяет получить улучшенные показатели качества в системах, изначально характеризующихся негативным влиянием межканальных связей на общую динамику системы. Эффективность предлагаемого подхода проиллюстрирована результатами численного моделирования, выполненного в среде MATLAB/Simulink.

Ключевые слова: адаптивное управление; гармонические возмущения; компенсация возмущений; наблюдатель Люенбергера; метод Фальба–Воловича.

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Introduction

In industrial practice, systems are typically designed as multi-input multi-output (MIMO) configurations, where complex interactions between control channels frequently occur. To address these interactions, decoupling strategies have been introduced as a reliable approach to suppress undesired cross-effects between inputs and outputs. The objective of the decoupling process is to ensure that each output is influenced only by its corresponding input, thereby enabling independent control among the channels in the system.

Decoupling methods are often divided into two main categories [1]: static and dynamic approaches. Static decoupling typically involves the use of gain matrices and is considered easy to implement, but such methods are generally effective only under steady state conditions. On the other hand, dynamic approaches, including ideal decoupling, simplified schemes, and those based on matrix inversion, are designed to manage both transient and steady state behaviors. Despite their broader capabilities, these dynamic methods tend to be more complex, more sensitive to model errors, and less robust when dealing with transmission zeros in the right half of the complex plane [2]. To overcome these limitations, many dynamic decoupling strategies using state feedback have been developed [3–5]. Among these, the method based on the Falb Wolovich approach [5] has been found to combine implementation simplicity with strong performance, even when model uncertainties and external disturbances are present [6].

The paper focuses on the problem of adaptive disturbance compensation in tracking control for a class of linear time-invariant multi-input multi-output systems, when the vector of output variables is measurable only, while the state vector is not accessible for measurements. The reference signal and external disturbances considered in this study are both unknown and take a harmonic form, and thus can be regarded as unknown disturbances acting on the control system. An effective approach to disturbance compensation is based on the internal model principle (IMP), in which the disturbance is modeled as the output of a linear autonomous system. To achieve zero steady-state error, the state and parameters of this system must be incorporated into the control law.

The problem of adaptive disturbance compensation based on the internal model principle is well known, particularly in cases where the disturbance is modeled as the output of an exosystem with unknown parameters (see Section 1.4 of [7, 8]). The objective of this approach is to design an adaptive control law in the form of either state feedback (see Section 4.2 of [7, 9, 10]) or output feedback [8, 11]. This method has been successfully developed for both SISO systems [12–16] and MIMO systems (see Section 4.2 of [7, 9, 10]). All of these studies assume that the system has a measurable state vector. In relation to this issue, the most recent publication [17] addressed the output-based problem for systemshowever, the disturbances acting on the system are required to be matched within the system.

This study is developed based on the work on adaptive output regulation [7] (see Section 4.2.3 in [7]). In this paper, we propose a new method for the direct compensation of unknown external disturbances in the tracking control problem for a class of unstable linear MIMO systems, where the external disturbances may be unmatched within the system structure. The proposed approach begins with the construction of a Luenberger observer to estimate the system states. Then, a linear decoupling controller based on state feedback is applied to eliminate undesired cross-coupling effects between inputs and outputs. Finally, the control law and the adaptive algorithm are designed using the estimated states from the reference observer and the disturbance observer of the decoupled system, ensuring that the system output accurately tracks the reference signal. The main advantage of the proposed algorithm is that the adaptive law becomes easier to tune due to the application of the decoupling strategy. Moreover, since the decoupled structure enables independent adjustment of the adaptation gains in each channel, the designer can properly select these gains to balance the convergence speed and robustness according to the desired performance requirements. In addition, improved control performance can be achieved in situations where the coupling between the channels of the original system adversely affects the overall system dynamics. Typically, the presence of inter-channel connections in MIMO systems leads to an increase in the overshoot; decoupling the system channels helps reduce the negative impact of these interconnections.

The paper is organized as follows. In Section 1, the problem is mathematically formulated, and the key assumptions are introduced. Section 2 presents the decoupling control strategy based on the Falb-Wolovich approach and the construction of the Luenberger state observer. The design of the disturbance observer and the reference observer is described in Section 3. The synthesis of the control law along with the adaptive algorithm is presented in Section 4. Section 5 provides numerical simulation results obtained using the Matlab software.

Notation: $\| x \|$ is the Euclidean norm of a vector x; $\mathrm{Col}(x_i)$ is a column vector with elements x_i; $blkdiag\{x_i\}$ is the block-diagonal matrix with diagonal elements x_i; $diag\{x_i\}$ is the diagonal matrix with diagonal elements x_i.

1. Problem statement

Consider the system described in the form:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Dv(t), \\ y(t) = Cx(t) + Ev(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^m$ and $v(t) \in \mathbb{R}^k$ denote the state vector, control input vector, output vector, and external disturbance vector, respectively. The matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$ and $D \in \mathbb{R}^{n\times k}$ are known, while matrix A may be unstable, the matrix $E \in \mathbb{R}^{m\times k}$ may be unknown, and the dimensions satisfy $m \le n$.

The control objective is to design a state feedback control law such that the following requirement is satisfied:

$$\lim_{t\to\infty} \| y(t) - g(t) \| = 0. \tag{2}$$

Here, $g(t) \in \mathbb{R}^m$ is the reference signal.

For this paper, the following assumptions are considered:

Assumption 1. The system is minimum phase, and the triplet (A, B, C) is controllable and observable.

Assumption 2. The vector of output variables y is measurable only, while the state vector x is not accessible for measurements.

Assumption 3. The reference signal $g(t) = [g_1(t), g_2(t), \ldots, g_m(t)]^\top$ is measurable. Each component $g_i(t)$, $i = 1, 2, \ldots, m$. is generated by a linear autonomous system:

$$\dot{\xi}_{gi}(t) = \Gamma_{gi}\xi_{gi}(t), \qquad g_i(t) = h_{gi}^\top \xi_{gi}(t). \tag{3}$$

Here, the state vector $\xi_{gi}(t) \in \mathbb{R}^{q_{gi}}$ is not directly measurable. The matrix Γ_{gi} is constant and unknown, with simple eigenvalues located on the imaginary axis. The vector $h_{gi}^\top$ is also unknown but constant. Each pair (Γ_{gi}, h_{gi}) is observable, and the upper bound q_{gi} is known.

Assumption 4. The external disturbance $v(t) = [v_1(t), v_2(t), \ldots, v_k(t)]^\top$ is not measurable. Each component $v_\alpha(t)$, $\alpha = 1, 2, \ldots, k$. is generated by a linear autonomous system:

$$\dot{\xi}_{v\alpha}(t) = \Gamma_{v\alpha}\xi_{v\alpha}(t), \qquad v_\alpha(t) = h_{v\alpha}^\top \xi_{v\alpha}(t). \tag{4}$$

Here, the state vector $\xi_{v\alpha}(t) \in \mathbb{R}^{q_{v\alpha}}$ is not directly measurable. The matrix $\Gamma_{v\alpha}$ is constant and unknown, with simple eigenvalues located on the imaginary axis. The vector $h_{v\alpha}^{\top}$ is also unknown but constant. Each pair $(\Gamma_{v\alpha}, h_{v\alpha})$ is observable, and the upper bound $q_{v\alpha}$ is known.

Remark 1. From Assumptions 3 and 4, the reference signal and the external disturbance can be represented in a simplified multi-output system form.

The reference signal can be modeled as

$$\dot{\xi}_g(t) = \Gamma_g \xi_g(t), \qquad g(t) = H_g^{\top} \xi_g(t),$$

where

$$\Gamma_g = blkdiag\{\Gamma_{gi}\},\ H_g^{\top} = blkdiag\{h_{gi}^{\top}\},\ \xi_g(t) = [\xi_{g1}^{\top}(t)\cdots\xi_{gm}^{\top}(t)]^{\top} \in \mathbb{R}^{q_g},\ \Gamma_g = blkdiag\{\Gamma_{gi}\},$$

$$H_g^{\top} = blkdiag\{h_{gi}^{\top}\},\ \xi_g(t) = [\xi_{g1}^{\top}(t),\ldots,\xi_{gm}^{\top}(t)]^{\top} \in \mathbb{R}^{q_g},\ \text{and}\ q_g = \sum_{i=1}^{m} q_{gi}.$$

Similarly, the external disturbance can be modeled as

$$\dot{\xi}_v(t) = \Gamma_v \xi_v(t), \qquad v(t) = H_v^{\top} \xi_v(t),$$

where $\Gamma_v = blkdiag\{\Gamma_{v\alpha}\}$, $H_v^{\top} = blkdiag\{h_{v\alpha}^{\top}\}$, $\xi_v(t) = [\xi_{v1}^{\top}(t),\ldots,\xi_{v\alpha}^{\top}(t)]^{\top} \in \mathbb{R}^{q_v}$, and $q_v = \sum_{\alpha=1}^{k} q_{v\alpha}$.

2. Design of the decoupler and the state observer

Consider the state-space representation of the system (1) by neglecting the effect of external disturbances

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu_d(t), \\ y(t) = Cx(t). \end{cases} \tag{5}$$

Here, $u_d(t)$ is the control law that decouples the system.

Since direct access to the full state vector $x(t)$ is not feasible, a Luenberger observer [18] is designed to estimate the state of system (5). The corresponding observer structure is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu_d(t) + L\left(y(t) - C\hat{x}(t)\right), \tag{6}$$

where $\hat{x}(t)$ represents the estimated state vector, and $L \in \mathbb{R}^{n\times m}$ denotes the observer gain matrix.

A control input $u_d(t)$ is constructed as in [5] in order to achieve input-output decoupling for the system (5), and is defined by the relation:

$$u_d(t) = -K\hat{x}(t) + Fu(t), \tag{7}$$

where K and F are constant real-valued matrices with compatible dimensions.

Let the output matrix C can be written as $C = [c_1, c_2, \ldots, c_m]^{\top}$, where c_i represents the ***i***-th row of the matrix C. For every $i = 1, 2, \ldots, m,$ we define a positive integer σ_i, known as the output order difference, as follows:

$$\sigma_i = \begin{cases} \min\left(l \mid c_i A^{l-1} B \neq 0^{\top}\right), & l = 1, 2, \ldots, n-1, \\ n-1, & \text{if } c_i A^{l-1} B = 0^{\top}, l = 1, 2, \ldots, n. \end{cases} \tag{8}$$

Here, $0^{\top} \in \mathbb{R}^m$ denotes a row vector whose entries are all zero.

The target transfer functions for the SISO components, obtained from the decoupled structure of system (5), can be described as

$$W_{ii}(s) = \frac{q_{i0}}{q_{i0} + q_{i1}s + q_{i2}s^2 + \cdots + s^{\sigma_i}} = \frac{q_{i0}}{\prod_{j=1}^{\sigma_i}(s - \lambda_{ij})},$$

where s denotes the Laplace variable and λ_{ij} are the assigned eigenvalues corresponding to index pairs $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, \sigma_i$.

Let us define the matrices B^* and C^* as follows:

$$B^* = \begin{bmatrix} c_1 A^{\sigma_1 - 1} B \\ c_2 A^{\sigma_2 - 1} B \\ \vdots \\ c_m A^{\sigma_m - 1} B \end{bmatrix}, \quad C^* = \begin{bmatrix} c_1 A^{\sigma_1} + \sum_{z=0}^{\sigma_1 - 1} q_{1z} c_1 A^z \\ \vdots \\ c_m A^{\sigma_m} + \sum_{z=0}^{\sigma_m - 1} q_{mz} c_m A^z \end{bmatrix}.$$

Based on these definitions, we can formulate the following result.

Theorem 1.

The system described in (5) can be decoupled if and only if the matrix B^* is invertible. Under this condition, the gain matrices in the control expression (7) are determined as $K = (B^*)^{-1} C^*$, $F = (B^*)^{-1}$. Consequently, the overall transfer function matrix of the system becomes diagonal and can be written as $W_u(s) = diag\{W_{ii}(s)\}$.

For the proof see Chapter 4 of [1].

Remark 2. Following the decoupling of system (1) via the control strategy defined in (7) using the matrices *K* and *F* from Theorem 1, the resulting structure corresponds to *m* separate SISO subsystems. The overall system can then be expressed in the state-space form:

$$\begin{cases} \dot{x}(t) = A_\Sigma x(t) + B_\Sigma u(t) + Dv(t), \\ y(t) = Cx(t) + Ev(t), \end{cases} \tag{9}$$

where the matrices $A_\Sigma = A - BK$ and $B_\Sigma = BF$ describe the decoupled dynamics. The transfer matrix $W_u(s) = C(sI - A_\Sigma)^{-1} B_\Sigma$ has a diagonal form, and A_Σ is Hurwitz.

3. Design of the reference observer and the disturbance observer

The reference input and external disturbance can both be expressed using their canonical representations, as discussed in [7, 19, 20]. These canonical representations serve as the basis for constructing the reference observer and the external disturbance observer.

The exosystem described in (3) can be transformed into the following canonical form:

$$\dot{\bar{\xi}}_{gi}(t) = G_{gi} \bar{\xi}_{gi}(t) + l_{gi} g_i(t), \qquad g_i(t) = \theta_{gi}^\top \bar{\xi}_{gi}(t), \tag{10}$$

where $\bar{\xi}_{gi}(t) \in \mathbb{R}^{q_{gi}}$ (for $i = 1, 2, \ldots, m$) denotes the state vector with initial value $\bar{\xi}_{gi}(0)$, $\theta_{gi} \in \mathbb{R}^{q_{gi}}$ represents a parameter vector that is unknown; $G_{gi} \in \mathbb{R}^{q_{gi} \times q_{gi}}$ is a Hurwitz matrix; and l_{gi} is a constant vector ensuring the controllability of the pair (G_{gi}, l_{gi}).

The exosystem given in (10) can alternatively be written in a block-diagonal structure as:

$$\dot{\bar{\xi}}_g(t) = G_g \bar{\xi}_g(t) + L_g g(t), \qquad g(t) = \theta_g^\top \bar{\xi}_g(t), \tag{11}$$

or equivalently:

$$\dot{\bar{\xi}}_g(t) = (G_g + L_g \theta_g^\top) \bar{\xi}_g(t), \qquad g(t) = \theta_g^\top \bar{\xi}_g(t), \tag{12}$$

where

$$\bar{\xi}_g(t) = [\bar{\xi}_{g1}^\top(t), \bar{\xi}_{g2}^\top(t), \ldots, \bar{\xi}_{gm}^\top(t)]^\top \in \mathbb{R}^{q_g}; \quad \theta_g^\top = blkdiag\{\theta_{gi}^\top\} \in \mathbb{R}^{m \times q_g};$$

$$G_g = blkdiag\{G_{gi}\} \in \mathbb{R}^{q_g \times q_g}; \quad L_g = blkdiag\{l_{gi}\} \in \mathbb{R}^{q_g \times m}.$$

Since the signal $g(t)$ is directly measurable, equation (11) allows the construction of a reference observer as follows:

$$\dot{\hat{\xi}}_g(t) = G_g \hat{\xi}_g(t) + L_g g(t), \qquad g(t) = \theta_g^\top \hat{\xi}_g(t) + \epsilon_r, \tag{13}$$

where $\hat{\xi}_g(0)$ is initialized arbitrarily. Then, the true state $\bar{\xi}_g(t)$ can be expressed as:

$$\bar{\xi}_g(t) = \hat{\xi}_g(t) + \epsilon_g,$$

with the estimation errors ϵ_r and ϵ_g decaying exponentially over time.

Similar to the reference signal, the exosystem described in (4) can be transformed into the following canonical form:

$$\dot{\bar{\xi}}_{v\alpha}(t) = G_{v\alpha}\bar{\xi}_{v\alpha}(t) + l_{v\alpha}v_\alpha(t), \qquad v_\alpha(t) = \theta_{v\alpha}^\top \bar{\xi}_{v\alpha}(t), \tag{14}$$

where $\bar{\xi}_{v\alpha}(t) \in \mathbb{R}^{q_{v\alpha}}$ (for $\alpha = 1, 2, \ldots, k$) denotes the state vector with initial value $\bar{\xi}_{v\alpha}(t)$; $\theta_{v\alpha} \in \mathbb{R}^{q_{v\alpha}}$ represents a parameter vector that is unknown; $G_{v\alpha} \in \mathbb{R}^{q_{v\alpha}\times q_{v\alpha}}$ is a Hurwitz matrix, and $l_{v\alpha}$ is a constant vector ensuring the controllability of the pair $(G_{v\alpha}, l_{v\alpha})$.

The exosystem given in (14) can alternatively be written in a block-diagonal structure as:

$$\dot{\bar{\xi}}_v(t) = G_v\bar{\xi}_v(t) + L_v v(t), \qquad v(t) = \theta_v^\top \bar{\xi}_v(t), \tag{15}$$

or equivalently:

$$\dot{\bar{\xi}}_v(t) = \left(G_v + L_v\theta_v^\top\right)\bar{\xi}_v(t), \qquad v(t) = \theta_v^\top \bar{\xi}_v(t), \tag{16}$$

where

$$\bar{\xi}_v(t) = [\bar{\xi}_{v1}^\top(t), \bar{\xi}_{v2}^\top(t), \ldots, \bar{\xi}_{vk}^\top(t)]^\top \in \mathbb{R}^{q_v}; \quad \theta_v^\top = blkdiag\{\theta_{v\alpha}^\top\} \in \mathbb{R}^{k\times q_v};$$
$$G_v = blkdiag\{G_{v\alpha}\} \in \mathbb{R}^{q_v\times q_v}; \quad L_v = blkdiag\{l_{v\alpha}\} \in \mathbb{R}^{q_v\times k}.$$

Following equation (15), the disturbance observer proposed in [7, 22] is constructed to estimate the state $\bar{\xi}_v(t)$. In this approach, the actual system state $x(t)$ is replaced by its estimate $\hat{x}(t)$ which is generated via the Luenberger observer (6). The disturbance observer is constructed in the following structure:

$$\dot{\varphi}(t) = G_v\varphi(t) + \left(G_v N - NA_\Sigma\right)\hat{x}(t) - NB_\Sigma u(t), \qquad \hat{\xi}_v(t) = \varphi(t) + N\hat{x}(t), \tag{17}$$

where the initial value $\varphi(0)$ is arbitrary. The matrix $N \in \mathbb{R}^{q_v\times n}$ is selected such that it satisfies the condition

$$ND = L_v. \tag{18}$$

Then $\bar{\xi}_v(t) = \hat{\xi}_v(t) + \epsilon_v$, where $\hat{\xi}_v(t)$ denotes the estimate of $\bar{\xi}_v(t)$, while ϵ_v exponentially decays.

Remark 3. Based on equations (17) and (18), it follows that each element of the disturbance vector v can be represented by a separate exosystem. The matrices G_v and L_v, due to their block-diagonal form, allow equations (16) and (17) to be rewritten as a collection of k independent subsystems:

$$\hat{\xi}_{v\alpha}(t) = \varphi_\alpha(t) + N_\alpha\hat{x}(t), \qquad \dot{\varphi}_\alpha(t) = G_{v\alpha}\varphi_\alpha(t) + (G_{v\alpha}N_\alpha - N_\alpha A_\Sigma)\hat{x}(t) - N_\alpha B_\Sigma u(t),$$

for each $\alpha = 1, 2, \ldots, k.$ The matrices $N_\alpha \in \mathbb{R}^{q_{v\alpha}\times n}$ are chosen such that the condition below is satisfied:

$$N_\alpha D = [0_{q_{v\alpha}}, \ldots, 0_{q_{v\alpha}}, l_{v\alpha}, 0_{q_{v\alpha}}, \ldots, 0_{q_{v\alpha}}],$$

where the vector $l_{v\alpha}$ is the α-th column.

4. Synthesis of the control law and adaptation algorithm

The decoupling process transforms system (1) into the form described in (9). In practice, disturbances may not be properly matched within the system. Therefore, to convert system (9) into a form in which the reference signal and external disturbances are compatible with the control input $u(t)$, we use the matrix regulator equation [7, 23, 24]. According to Assumptions 3 and 4, the eigenvalues of the matrices Γ_g and Γ_v or, equivalently, those of $(G_g + L_g\theta_g^\top)$ and $(G_v + L_v\theta_v^\top)$ are ensured not to coincide with the transmission zeros of the transfer matrix $W_u(s)$. Therefore, for system (9), the following results are obtained. There exist matrices $M_g \in \mathbb{R}^{n\times q_g}$ and $\eta_g \in \mathbb{R}^{q_g\times m}$ such that the matrix equalities

$$B_\Sigma \eta_g^\top = A_\Sigma M_g - M_g (G_g + L_g \theta_g^\top), \qquad CM_g = \theta_g^\top. \tag{19}$$

are satisfied. Similarly, there exist matrices $M_v \in \mathbb{R}^{n\times q_v}$, $\eta_v \in \mathbb{R}^{q_v \times k}$ such that the following equalities hold:

$$B_\Sigma \eta_v^\top = A_\Sigma M_v - M_v (G_v + L_v \theta_v^\top) + D\theta_v^\top, \qquad CM_v = -E\theta_v^\top. \tag{20}$$

Remark 4. We observe that both matrix $\theta_g^\top$ and matrix $\theta_v^\top$ are unknown. Consequently, the matrix pairs (M_g, η_g) and (M_v, η_v) are also unknown. However, in this case, it is sufficient to know that such matrix pairs exist.

We define the state error as $e(t) = x(t) - M\bar{\xi}(t)$, and the output tracking error as $\delta(t) = y(t) - g(t)$, where $M = [M_g, M_v]$ and $\bar{\xi}(t) = [\bar{\xi}_g(t), \bar{\xi}_v(t)]^\top$. Then, taking into account equations (9), (12), and (16), we compute the time derivative of $e(t)$ and obtain:

$$\dot{e}(t) = A_\Sigma e(t) + B_\Sigma u(t) + (A_\Sigma M_g - M_g(G_g + L_g\theta_g^\top))\bar{\xi}_g(t) + (A_\Sigma M_v - M_v(G_v + L_v\theta_v^\top) + D\theta_v^\top)\bar{\xi}_v(t) + \epsilon .$$

Based on the matrix equations (19) and (20), the system can be represented in the form:

$$\dot{e}(t) = A_\Sigma e(t) + B_\Sigma(\eta^\top \bar{\xi}(t) + u(t)) + \epsilon, \qquad \delta(t) = Ce(t).$$

Here, the matrix $\eta^\top = [\eta_g^\top, \eta_v^\top]$ is unknown.

Based on (13) and (17), and neglecting the exponentially decaying term due to the stability of $W_u(s)$, the state $\bar{\xi}(t)$ can be replaced by its estimate $\hat{\xi}(t)$. Consequently, the system can be rewritten in the form

$$\dot{e}(t) = A_\Sigma e(t) + B_\Sigma[\eta^\top \hat{\xi}(t) + u(t)], \qquad \delta(t) = Ce(t). \tag{21}$$

Since the matrix A_Σ is Hurwitz, the control law for system (21) can be formulated as follows:

$$u = -\hat{\eta}^\top \hat{\xi}(t). \tag{22}$$

Here, the matrix $\hat{\eta}^\top = [\hat{\eta}_1^\top(t), \hat{\eta}_2^\top(t), \ldots, \hat{\eta}_m^\top(t)] \in \mathbb{R}^{m\times q}$ is determined using an adaptive algorithm.

As a result, the system described in equation (21) can be reformulated as

$$\delta(t) = W_u(s)[\tilde{\eta}^\top \hat{\xi}(t)], \tag{23}$$

where $\tilde{\eta}^\top = \eta^\top - \hat{\eta}^\top$ is the matrix of the adjustable parameters.

Based on model (23), a gradient-based adaptive algorithm can be applied to estimate the tuning parameter. However, this algorithm exhibits poor convergence performance and may not allow acceleration of convergence even when the regressor satisfies the persistent excitation condition [25]. In this study, we employ an adaptive algorithm with memory regressor extension (MRE) [7] to improve the convergence rate of signals in the closed-loop system.

Proposition 1. Under Assumptions 1 to 4, if system (1) satisfies the decoupling condition in Theorem 1, then by applying the adaptive algorithm with MRE to the decoupled system (9), the adaptation gain can be adjusted independently for each control channel.

Proof:

From equation (23), we define the extended error variable

$$\hat{\varepsilon} = \delta(t) + \Delta^\top \hat{\eta}, \tag{24}$$

Where

$$\Delta^\top = W_u(s)[\hat{\xi}^\top(t)] = \begin{bmatrix} W_{11}(s)[\hat{\xi}^\top(t)] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & W_{mm}(s)[\hat{\xi}^\top(t)] \end{bmatrix}. \tag{25}$$

From (25), we see that each diagonal element is a regressor filtered through the dynamics of channel i:

$$\Delta_i^\top = W_{ii}(s)[\hat{\xi}^\top(t)], \qquad i = 1, 2, \ldots, m.$$

By taking into account (23), equation (24) is rewritten:

$$\hat{\varepsilon} = \Delta^\top \eta. \tag{26}$$

We multiply both sides of equation (26) by $\Delta = W_u(s)[\hat{\xi}(t)]$ and applying the transfer function $H(s) = \dfrac{1}{\beta s + 1}$ with $\beta > 0,$ we obtain the following expression:

$$H(s)[\Delta\hat{\varepsilon}] = H(s)[\Delta\Delta^\top]\eta, \tag{27}$$

Based on equation (27), the adaptive algorithm is constructed as follows:

$$\dot{\hat{\eta}} = \mu(Y - \Omega\hat{\eta}), \tag{28}$$

where $Y = H(s)[\Delta\hat{\varepsilon}], \Omega = H(s)[\Delta\Delta^\top], \mu > 0.$ is the adaptation gain.

Since the vector Y and the matrix Ω are constructed from a the matrix Δ matrix with a diagonal structure, Y and Ω can therefore be represented in the following form:

$$Y = \mathrm{Col}(H(s)[\Delta_i\hat{\varepsilon}_i]), \text{ for } i = 1, 2, \ldots, m; \tag{29}$$

$$\Omega = blkdiag(H(s)[\Delta_i\Delta_i^\top]), \text{ for } i = 1, 2, \ldots, m. \tag{30}$$

From expressions (28), (29), and (30), we can present the adaptive algorithm for each control channel in the form:

$$\dot{\hat{\eta}}_i = \mu_i(Y_i - \Omega_{ii}\hat{\eta}_i), \tag{31}$$

where $Y_i = H(s)[\Delta_i\hat{\varepsilon}_i], \quad \Omega_i = H(s)[\Delta_i\Delta_i^\top], \quad i = 1, 2, \ldots, m.$

Based on expression (31), Proposition 1 has been proved.

Remark 5. The adaptive algorithm in the form of (31) ensures that the parameters between channels are no longer interdependent, and each parameter only needs to be adjusted based on the error of its own channel. As a result, the convergence process for each channel becomes independent, and the adaptive algorithm becomes easier to tune.

By employing the adaptation algorithm (31), it can be concluded that the control objective (2) is achievable [7, 26] with the proposed method, and the following theorem can be stated.

Theorem 2. Suppose that Assumptions 1 to 4 are satisfied. Then, the state observer (6) applied to system (5), and the control law (22) together with the reference observer (13), the disturbance observer (17) and (18), and the adaptation algorithm (31), when applied to system (9), which is obtained from system (1) by applying the control law (7) with the matrices introduced in Theorem 1, ensure that all signals in the closed loop system are bounded and the control objective (2) is achieved.

5. Numerical example

We consider a third-order unstable linear system (1) with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0{,}1 & -0{,}9 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ -1 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}.$$

Suppose the reference signal $g_1(t) = 3\sin(2t)$ is a harmonic signal characterized by unknown parameters such as frequency, amplitude, phase, and bias. Additionally, let $g_2(t) = 2$ be an unknown constant. Under this assumption, $g_1(t)$ can be represented using a second-order exosystem as described in (10), whereas $g_2(t)$ can be represented by a first-order exosystem. Therefore, the reference observer (13) is constructed using the matrices

$$G_{g1} = \begin{bmatrix} 0 & 1 \\ -6 & -7 \end{bmatrix}, \quad l_{g1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad G_{g2} = -9, \quad l_{g2} = 1, \quad G_g = \begin{bmatrix} 0 & 1 & 0 \\ -6 & -7 & 0 \\ 0 & 0 & -9 \end{bmatrix}, \quad L_g = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Similarly, the external disturbance is assumed to have the form $v_1 = 3\sin(2t), \quad v_2 = 3\sin(3t) + 2.$ Therefore, the disturbance observer (17)–(18) is constructed using the matrices

$$G_{v1} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}, \quad l_{v1} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad G_{v2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix},$$

$$l_{v2} = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -3 & 3 & 6 \end{bmatrix}.$$

In designing the Luenberger observer, the eigenvalues –2, –3 and –4 are selected. Using the pole placement technique, the observer gain matrix L is calculated as:

$$L = \begin{bmatrix} 16{,}1 & 0 \\ 57{,}61 & 0 \\ -8 & 8 \end{bmatrix}.$$

Based on equation (8), we determine $\sigma_1 = 2$ and $\sigma_2 = 1$ for the respective outputs. Choosing the poles for the first channel as $\lambda_{11} = -2, \lambda_{12} = -3$ and for the second channel as $\lambda_{21} = -4$, the decoupling control law (7) is designed using the following matrices:

$$F = \begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}, \quad K = \begin{bmatrix} 6{,}1 & 4{,}1 & 0 \\ 4 & 1 & 5 \end{bmatrix}.$$

For the adaptation algorithm with MRE (31), we choose $H(s) = \dfrac{1}{s+1}$.

The simulation results are presented in Figures 1 and 2 to verify the effectiveness of the proposed approach. In Figure 1, we simulate the algorithm with the adaptation gain for channel 1 set to $\mu_1 = 1$ and for channel 2 set to $\mu_2 = 0{,}5$, whereas in Figure 2 we simulate with the adaptation gain for channel 1 set to $\mu_1 = 3$ and for channel 2 set to $\mu_2 = 1$.

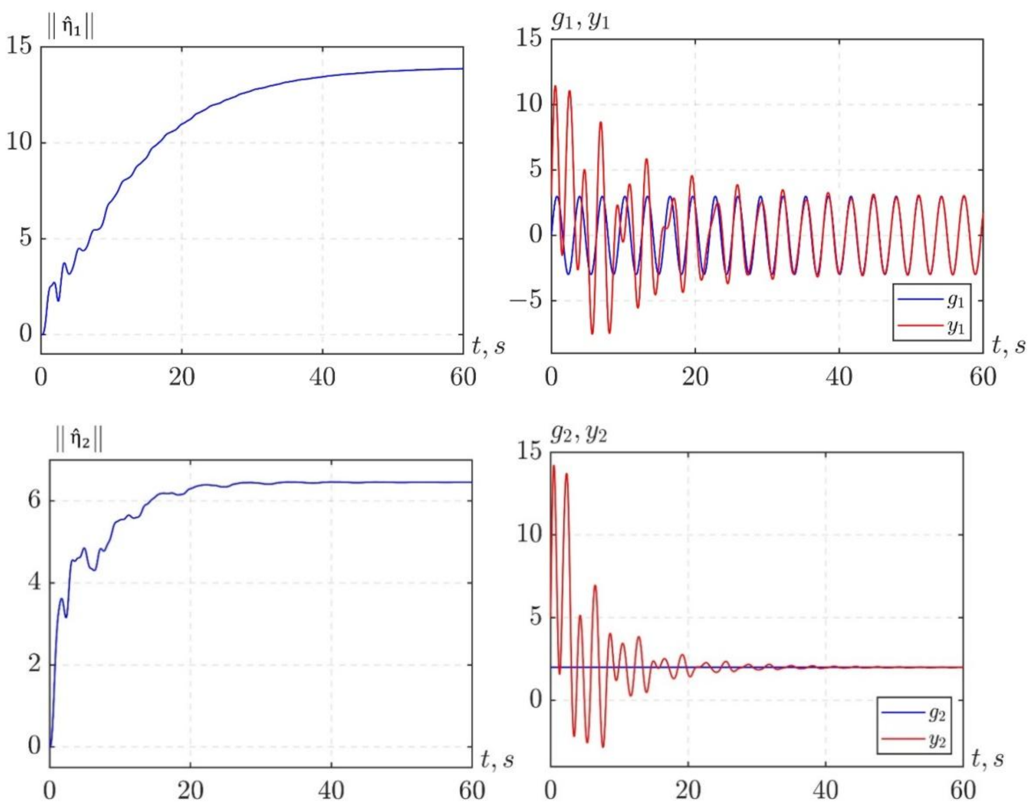

Fig. 1. Transients in the closed-loop control system with $\mu_1 = 1$ and $\mu_2 = 0{,}5$

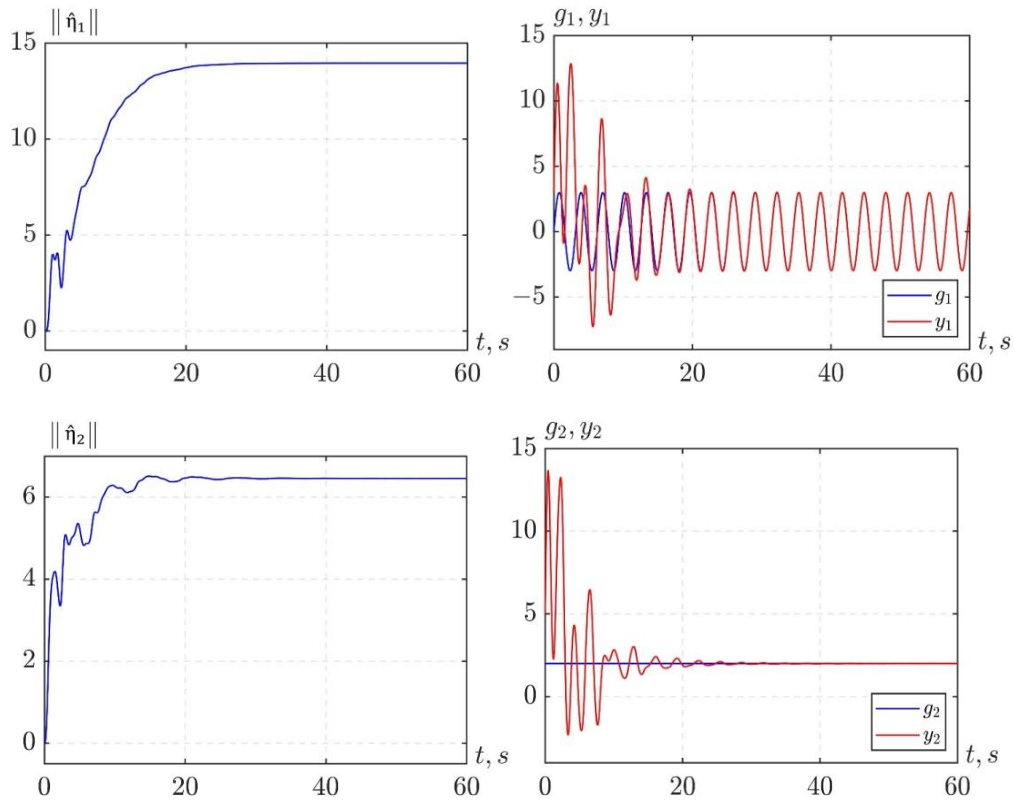

Fig. 2. Transients in the closed-loop control system with $\mu_1 = 3$ and $\mu_2 = 1$

By observing the simulation results, we can see that the control objective (2) is achieved and the proposed algorithm allows independent adjustment of the adaptation gains between the channels. Comparing Figures 1 and 2, we observe that the convergence speed increases when the adaptation gains are increased. Moreover, we also see that in this case the convergence is fast even with small adaptation gains.

Conclusion

This paper presents the development of the method for adaptive disturbance compensation in linear multivariable systems subjected to unknown multi-harmonic disturbances presented in [7]. The application of a system decoupling strategy based on the Falb–Wolovich approach makes the adaptive law easier to tune due to the independent adjustment of the adaptation gains. By properly selecting these gains, the designer can balance the convergence speed and robustness according to the required performance specifications. In addition, it is possible to get improved control performance in situations where negative inter-channel coupling of an original system has an adverse effect to the overall dynamic of the system. In the future, the authors plan to extend this method to address the problem of output-based disturbance compensation for systems with multiple input delays.

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