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**POINCARÉ AND THE ANALYTIC TRADITION<sup>1</sup>**

*There is no substantive agreement about the philosophical views of analytic philosophers. Nevertheless, for much of them logical analysis is widely recognized to be important. It is true that Poincaré used no logical analysis but refused nevertheless the old metaphysics. Indeed, the analytic tradition of philosophy of science is perhaps better characterized by several overlapping similarities, which are the clincher for my main thesis: Analytic philosophy of science has one of its origins in the philosophical network in France around 1900 and, especially, in Poincaré.*

**Keywords:** *Poincaré, Analytic Philosophy of Science, Logical Empiricism, Mathematics, French philosophy, Logic.*

**Summary**

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**I. Introduction**

In this paper I ask the general question: Are there systematic and/or historic relations between French philosophy of science and the analytic tradition? My answer and main thesis will be: yes ! Analytic philosophy of science has one of its origins in the philosophical network in France around 1900.

Of course, all depends on what is meant by 'Analytic Philosophy of Science'. Clearly, the historical proponents of analytic philosophy, Frege and Russell, did not influence Poincaré in a positive way and Poincaré used no logical analysis but refused nevertheless the old metaphysics: "those who consider metaphysics, since

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<sup>1</sup> This paper draws on three talks given at the University of Lorraine, at the University of Tomsk and at the University of San Francisco. I am indebted to David Stump, University of San Francisco, for his help in correcting my English. The work on this paper was carried out as part of the ANR / DFG project *Mathematics: Objectivity by Representation*.

Auguste Comte, as old-fashioned, tell me that there can be no modern metaphysics. But the negation of all metaphysics, it is still a metaphysics, and this is precisely what I call modern metaphysics" [1; cf. 2. P. 79 and 3. P. 34]. Regarding conceptual analysis, mathematicians had already learned at the end of the 19th century what analytic philosophers attained only in 1950: Weierstrass' monsters (non-differentiable continuous functions) and other monsters, so contrary to common sense, "could not be blamed on poorly understood notions, as in the time of indivisibles" [4. P. 27]. Rather, the logical conceptual analysis created these counter-intuitive facts! Poincaré himself was surprised that intuition can us "deceive at this point" [5. P. 200]. While mathematicians have been for a long time conscious that conceptual analysis is *per se* not sufficient, if not guided by an explicit insight, philosophers were only conscious of this fact by the criticisms of Carnap's language program.

There is now substantive agreement about the philosophical views of analytic philosophers. Nevertheless, for much of them logical analysis is widely recognized to be important. Analytic philosophy is by no means a 'school'! Indeed, the analytic tradition of philosophy of science is perhaps characterized by overlapping similarities, which run as follow:

- I. Put up a thesis and try to argue for it by considering
  - a) the arguments of the field's specialists,
  - b) the conceptual presuppositions and the linguistic level
  - c) the methodological/metaphysical presuppositions (realist/ant-realist, pragmatist etc.)
  - d) technical tools (context dependency: logic, cognitive sciences etc).
- II. Rationality is based on interaction and belief revision.
- III. There is a core of analytic philosophers who should be included as historical perspective of I-II.

These criteria are sufficient to make Peirce an analytic philosopher of science and to exclude Bergson! Now, Henri Poincaré is the main figure of a French Network around 1900. Can he be seen as a philosopher in the analytic tradition? If yes, does it matter whether Poincaré was analytic philosopher?

Immediately, there is a first doubt: Is Poincaré a philosopher? He was obviously a mathematician, an astronomer and a physicist, but a philosopher? According to Karl Popper, he was a very great philosopher, the greatest philosopher of science [6. P. 2]. Is this not an excessive judgment? Where is his philosophical work? Is it not exhausted in four or five collections of articles? How can he be compared to other scholars whose names evoke, beyond any doubt, the quality of "very great philosopher" such as Locke, Comte, Peirce, Bergson, Russell, or Quine?

Indeed, what does it mean to be a philosopher? It's not to be a scientist: "in philosophy there are no proofs; there are no theorems; and there are no questions which can be decided, Yes or No" [7. P. 345]. Under the guise of theory of knowledge – which alone is pertinent for our purpose – I propose the following answer: the philosopher discusses logically compelling the decision problem concerning the conceptual framework of the knowledge of scientific 'objects'. Thus, the philosopher should

– devise the conceptual presuppositions that are at the root of our scientific knowledge, and

– clarify – in the sense of giving a deeper insight – this activity in its symbolic form.

In this sense, Poincaré is a philosopher! For Poincaré it necessitates a special sort of intuition, that is a « faculty which makes us see the end from afar » [5. P. 205]. More precisely, I think Poincaré's philosophical approach must be ranked among these sources of *analytic tradition*, that seem to be at the same time sources of Quine's criticism of the two dogmas of the logical empiricism: the dogma of the separation of observational and theoretical language and the dogma of the possible reduction of empirical meaning to experience. In fact, Quine denies that the distinction between analytic and synthetic concerns a logical difference and he even denies that a crucial experiment can determine the meaning of the observational terms.

We will see that Poincaré's conventional propositions are empirically underdetermined and neither analytic nor synthetic and, consequently, Poincaré may survive logical empiricism. Now, this exactly could be an argument to use Poincaré and his polemics against Russell, Couturat, Cantor and Hilbert as important player in the fight against analytic philosophy. It was and is the case in France. Indeed, such an interpretation leads to a potentially misleading understanding of his philosophical insights and is the reason why my main question is worth addressing not only from the historical point of view. The following remarks on Wittgenstein of Hans Jochim Glock, can be word by word transposed by substituting "Poincaré" for "Wittgenstein": "At present, there is also an increasing mutual isolation between Wittgenstein [Poincaré] scholarship and Wittgensteinian [Poincareian] philosophy, on the one hand, and mainstream analytic philosophy, on the other. In my view, this isolation is detrimental to both sides. Wittgenstein [Poincaré] presents us with highly original claims and arguments, which deserve to be taken seriously by contemporary analytic philosophers, since they challenge some of their basic assumptions. At the same time, Wittgenstein [Poincaré] scholarship and Wittgensteinian [Poincareian] philosophy can profit from reconstructing his ideas in an analytic fashion" [8. P. 420].

In his famous booklet *Origins of Analytic Philosophy*, Michel Dummett wrote:

"A grave historical distortion arises from a prevalent modern habit of speaking of analytical philosophy as 'Anglo-American'.... This terminology utterly distorts the historical context in which analytical philosophy came to birth, in the light of which it would better be called "Anglo-Austrian" than 'Anglo-American'." [9. P. 1–2]. Since Dummett's criterion to assign the name 'analytic philosophy' to an author's activity requires that he analyses thought by an analysis of language, a point not incompatible with Poincaré's view, I propose to substitute 'Anglo-Austrian-French' for 'Anglo-Austrian'. Surely, Dummett's criterion expresses just point I (b) in my list above and could "be accepted by important members of the hermeneutic and poststructuralist movements, such as Heidegger, Gadamer, and Derrida" [8. P. 428].

From a methodological point of view, I emphasize, in this paper, neither the historical context nor the mathematical stringency of Poincaré's argumentation. I am indeed interested in the question if Poincaré's writings give the conceptual possibility to classify him in 2016 as belonging to the analytic tradition. Whether the analytical tradition is considered as a sociological or philosophical phenome-

non, I think that Poincaré is closer to it than Bergson, Bachelard, Brunschvicg or Heidegger.

## **II. Poincaré as frontier commuters between Kantianism and Logical Empiricism**

The ambition to locate logical empiricism and hence the origin of analytic philosophy in the “inheritance” of Poincaré does not go without saying, because there exist two traditions of interpreting Poincaré’s work:

- one, which endorses his intuitionist tendency and at the same time his polemics against logicism or formalism (vulgate: “continental” tradition): “Poincaré strongly disagreed [with Russell and Frege], claiming that intuition was the life of mathematics” [10] and that his use of metaphors is a sign of continental vagueness.

- another, which opts for the conventional and linguistic aspects of his work that makes of him an initiator of the linguistic turn (vulgate: “analytic” tradition).

Is Poincaré a borderline case or can one decide to put Poincaré in one of these traditions? If yes, what is the criterion?

First of all, it is obvious and was never contested that Poincaré was strongly influenced by and integrated in a philosophical movement related to a mixture of empiricism and Neo-Kantism [11]. According to Poincaré, mathematics requires intuition not only in the context of discovery but also in the context of justification, especially in arithmetic and logic. So it is not surprising that Poincaré was even introduced in German philosophical circles by the Kantian tradition: examples are Moritz Schlick, Aloys Riehl and his follower Ilse Schneider.

Is Poincaré perhaps a neo-kantian? This interpretation would be too strong! The arithmetical “pure intuition” Poincaré introduces is intellectual (and not sensible) in character and Poincaré does not at all solve the problem of the unity of spontaneity and receptivity by the introduction of a pure sensibility. Rather, he changes the terms of the Kantian opposition, given that what is important for him is the balance between exactness and objectivity. The latter concerns a (intersubjective) consensus with respect to natural relations. Poincaré expresses the lost balance between exactness and objectivity in a formula well known by its popularization under Einstein’s pen: “what they [mathematics] have gained in rigor, they have lost in objectivity. It is by distancing themselves from reality that they acquired this perfect purity” [12. P. 446]. This is why Poincaré didn’t confine himself with the perfect purity.

Alberto Coffa noted that Schlick’s quandary sounds very analogous: “Explicit definition from given primitives gives representations that are linked to reality, but it can guarantee no more intersubjectivity than is available in its starting point. Since its starting point always consists of the subjective target of ostension (singular representation), it preserves the link with the reality at the price to give a solution [to the problem how to explain the rigorous character of scientific knowledge]. In contrast, implicit definition achieves sharpness, but the price is a complete lack of relation to the world” [13. P. 176]. In fact, the Kantian “heritage” of Logical empiricism was always very uncontroversial in Germany (where Paul Lorenzen and Wolfgang Stegmüller represented the German (but not Austrian) tradition of so diametrical opposed minds as Schlick and Carnap) and since Michael Friedman’s seminal book “Reconsidering Logical positivism” [14] it is even well established in

the States. As for neo-Kantian and Poincaré, the Kantian a priori-synthetic concept has given rise to *aporiae* which logical empiricism meant to overcome by postulating what Quine called the two dogmas, already mentioned above.

Given the fact that Logical Empiricism was influenced in a negative and positive way by the Kantian tradition, it should not be surprising that Poincaré was influenced by this tradition, too; or better: Poincaré's Kantian vocabulary is the expression of a non academic but nevertheless systematic philosopher and does not prevent reading his work from the point of view of Logical Empiricism.

It is equally known that, among those who were considered as "forerunners" of the logical empiricists, that the French philosopher of science Pierre Duhem distinguishes himself by the fact that he seems to be at the same time a forerunner of Quine's criticism. The Duhem-Quine holistic thesis concerns the calling in question of the separation of theoretical language from observational language.

Poincaré had not the chance to be mentioned so prominently by an Anglophone philosopher, however, he too must be ranked among these "forerunners" who survive Quine's criticism; not primarily for his holistic thesis but for his conception of geometrical conventions as a kind of bicephalous selection of analytical but non-logical propositions, "guided" at the same time by experience. His conventions were trivialized as purely linguistic decisions. In other words, it has but rarely been attempted to ascribe to the phrase "guided by experience" another meaning than that of a metaphor and it is probably not easy to do so without having first read the later Wittgenstein.

In order to explain my thesis more fully, I want introduce some methodological remarks. We may want to go on to ask questions like this: 'What would Poincaré have said about logical empiricism?'. But we shall not describe the answers we envisage him giving to such a question as descriptions of what he meant or did. This means that we are interested not only in what the Poincaré who walked the streets of Nancy could be brought to accept as correct description of what he had meant but in what an ideally reasonable and educable Poincaré could be brought to accept as such a description. You see that I propose a Rorty-like solution: rational reconstruction.

In a historical reconstruction we confine our investigations by reconstructing the historical context, arguing that the first duty of a philosopher is to understand the theses of historical persons with respect to their time.

But: we cannot find out what somebody means prior to finding out how his linguistic practice resembles and differs from ours. The assumption that historical reconstruction is naturally prior to rational reconstruction seems to be a rest of an insufficiently holistic account of interpretation [see 15. P. 55].

It is in terms of such a holistic account of interpretation, that I want argue for the thesis that Poincaré is a philosopher of analytic tradition: I try to interpret his metaphors in a clear way, despite of his often aphoristic and ironic style. It's not my question if in arguing as he did, he was always consistent (definitely, he was not), but to find out "what lies behind the system" he has built [see 7. P. 380].

### III. The Origins of Analytic Philosophy Revisited

#### (a) French Philosophy and Logical Empiricism

Here the path I intended to follow:

First, I will give an overview of some connections between French philosophy of science and its "extension" by "logical empiricists";

Secondly, I will present a reconstruction of Poincaré's "conventionalism" in geometry in order to evaluate in these lights his position with respect to logical empiricism.

Conventionalism was a main topic in the Polish group and especially studied by Kasimir Ajdukiewicz. As a 23 year old student in Göttingen (1913), he had been familiarized with Hilbert's and Poincaré's philosophy of mathematics.

Ajdukiewicz's so called "radical conventionalism" and its Poincareian background was analysed in a magisterial way by [16]. For Carnap, Ajdukiewicz's attempt "at a general syntactical investigation" was with Tarski's one of "the most important" forerunner of his own investigations [17. P. 16].

Leon Chwistek, too, was strongly influenced by Poincaré: not by the conventional but by the constructive aspect of his work. He develops type theory and rational semantics, which could be successfully applied to solving problems connected with philosophy, science, social theory, and art.

More generally, papers of Poincaré, Duhem and Federigo Enriques were read and discussed in the Circles and there is not only a direct exchange between Duhem and Mach but also an indirect exchange between Mach and Poincaré about the genesis of geometry.

In 1905, Mach's *Erkenntnis und Irrtum* appears. This book includes three chapters on the genesis of geometry [18, chap. 20, 21, 22] among which the last two are an explicit resumption of articles appeared also in the *Monist* in 1902 and 1903 [cf. 19. P.353. P. 389, notes 1]. On the advice of Mach himself the translator of *Erkenntnis and Irrtum* removes, in 1908, these two chapters in the French version, because they "would repeat what Mr Poincaré wrote on the question" [cf. 19. P. 247, notes 4].

Philippe Frank [20] planned, as early as 1907, to establish a synthesis of Mach's "economic descriptions of observed facts" and Poincaré's "free creations from human mind", which, together, he thought "was at the origin of what was later called logical empiricism".

The manifesto of the Vienna circle (1929) confirms these systematic and historical relations by echoing Poincaré's conventionalism [21. P. 112], Otto Neurath emphasis in 1931 that the Vienna Circle "seeks to create a climate which will be free from metaphysics" by continuing "the work of Mach, Poincaré, Frege, Wittgenstein and others" [22. P. 282] and, in their inauguration addresses of the Descartes Congress [23], Louis Rougier and Philipp Frank emphasize again the French relations to logical empiricism: Poincaré exercised a big influence on the groups of Vienna and of Prague where doctrines of Bergson, Meyerson and Boutroux were rejected [this is probably not correct with respect with Boutroux, G.H.]. Rougier (1889–1982) was one of the rare *French* philosophers who did not limit the *positive* reception of Poincaré's work to its psychologist and intuitionistic part but linked conventionalism and logic. In his 1913 paper entitled "Poincaré et la mort des vérités nécessaires" [24] he uses, as do later Schlick and [25. P. 144] Poincaré's famous Flatland model of the Lobachevskian geometry by supposing a world endowed with a peculiar temperature field in order to deduce the conventional character of physical geometry with respect to contingent circumstances of our milieu (i.e. suggested by experience) [26]. Further, Rougier sees this conventional character increased for Hilbert-type formalism of all sorts. This extension of

Poincaré's geometrical conventionalism to physics and its largely unhistorical interpretation was common to much of the logical empiricists, especially Moritz Schlick (1882–1936) and Rudolf Carnap.

The crowd of documents contained in Jules-Henri Greber's PhD thesis [27] shows that a movement in France around 1900 is related to the theses of the unified science: it is not only Poincaré's deliniation of hypotheses unifying the concept of assumptions for the different branches of science that creates this unification – it is certainly not ontological: in this sense the Aristotelian tradition is well respected – but also an important practical turn: discussions are conducted in magazines, the teaching of philosophy is enriched with a scientific background, the organization of Congresses internationalizes the movement. Louis Couturat who called Poincaré his "scholarly collaborator" is the main organizer of the First International Congress of Philosophy held in Paris in early August 1900 during which "the word 'epistemology' is used by Jean Wilbois (and then Russell) instead of the formula 'critic of science' or 'general criticism of science' .... in order to emphasize the philosophical reflection facing the growing powers of what we call today 'technosciences'" [28].

#### (b) Poincaré and revisions in the analytic tradition

Until the late 1980s, Frege was the main reference for the origins of both modern logic and analytic philosophy.

This perspective was corrected in four steps:

- (1) Jan Sebestic [29] introduces Bolzano;
- (2) Dummett [9] introduces Bolzano, Brentano, Meinong and Husserl;
- (3) Alberto Coffa's book entitled: *The Semantic Tradition from Kant to Carnap : To the Vienna Station* [13], introduces Néo-Kantism;
- (4) Amirouche Moktefi [30] established that Frege's work is not the first step of modern logic: an overlapping period (MacColl, Carroll etc.) should be recognized.

We add a fifth one: the Poincaré circle.

Thus one has to acknowledge different results of revision with respect to Poincaré's time, concerning the philosophical context and, of course, the technical development of logic. By considering Poincaré's own criticisms of modern formal logic, one could draw out the following points:

– Anti-psychologism was not only a Fregean endeavor but also an attempt of Bolzano, Husserl, Meinong and, partially, of Poincaré. If the meaning of a proposition is not to be situated in the consciousness, it seems that it has to be situated in a domain as different from the physical world as from the interior world of private events. Frege introduced for this reason a *third empire* different from social phenomena and Poincaré restricted his 'conventions', it is true, to special domains.

– The source of Logical Empiricism was not only the Fregean but also the neo-Kantian tradition. A misleading stereotypical characterization views logical empiricism as concerned to provide a philosophical justification of scientific knowledge without any historical considerations. But, in reality, logical empiricism follows very often the (Néo)-Kantian approach: its aim is rather to reform metaphysics in accordance with the already achieved success of the exact sciences [14. P. 4]. Poincaré, too, is involved in a philosophical tradition related to a mixture of empiricism and Neo-Kantism and his aim is to explain scientific progress.

– Tarski’s truth-conditions have to be supplemented by logical research on *understanding* truth (Bolzano). In the philosophico-mathematical tradition, the epistemic attitude (proving that something is true: “Gewissmachung”) has always prevailed over the etiological attitude (proving *why* something is true: “Begründung”) with the most notable exception of Bolzano. According to him, a proof must not only give the reason but also contain a semi-formal element to insure the understanding. Poincaré’s conception of *understanding* a proof echos this tradition.

– Classical first order logic is not sufficient (Poincaré). Hintikka’s independence-friendly (IF) logic (an improved version of the traditional Fregean first-order logic) is such a powerful tool that there is no longer any need to resort to set theory for the purpose of doing model theory of first order logic or of expressing complete induction. Consequently, Poincaré’s criticism concerning formal logic in its application to mathematics vanishes with the invention of IF logic. Indeed, far from being the only relevant development, the dialogical approach of logic conceives in the frame of Martin Löf’s constructive type theory a belief system in which the acquisition of knowledge (as an ideal limit) and the interactive aspects of meaning are perceived as a question-response game in respect to a set of initial hypotheses that are expressed at that very level in the object language. Belief is always a potential construction [31], so long as we consider the potential and the actual as very distinct and as Poincaré did. In this perspective, Poincaré’s criticism concerning nonconstructive set theory vanishes.

#### IV. Some general elements of Poincaré’s philosophy

*Thesis:* Poincaré always defends the same philosophy: it consists in a reconstruction program of the process of understanding scientific theories where the construction of scientific ‘objects’ is simultaneously conceived with the construction of language, or more exactly, where the empirical basis is the occasion of the process of language learning [see 32].

More precisely, Poincaré distinguishes *a priori* principles (for example, *mathematical induction* in arithmetic) from *conventions* as *apparent hypotheses* (for example, the parallel axiom in geometry) and from *true hypotheses* (for example, the hypothesis of central forces in physics). Placed side by side, all of these notions express local forms, specific to the different fields considered, of Poincaré’s philosophical problem, whose principle subject is (and I emphasize) the epistemological question of the relational form of scientific knowledge and its exact formulation.

In arithmetic, the counting process, conceived as an indefinite repetition, is occasioned by experience without being itself empirical and constitutes as such the theoretical part of our knowledge. Experience is the *ratio cognoscendi* of the *affirmation* of the fact that if a domain can be structured through an act of indefinite repetition, a property is valid for all elements if it is valid for the successor of any element. Thus, in arithmetic, we seem to use neither hypotheses nor conventions, but an operative intuition, i. e. not a cognitive capacity about entities but about what we are able to do (Wittgenstein).

One might argue that Poincaré was mistaken to include arithmetic in his book, if arithmetic is of non-hypothetical nature. I think he was in no way mistaken. What is important here is that this principle is understood to be a structural element of empirical investigations: “We have the faculty of conceiving that a unit may be

added to a collection of units. Thanks to experiment, *we have had the opportunity of exercising this faculty and are conscious of it*" [2. P. 25; my emphasis].

From this point of view, the principle of complete induction as a necessary tool of mathematics is very close to Poincaré's understanding of the principle of empirical induction. The latter is a *natural* hypothesis or a practical rule in the sense of a necessary tool of physics, which "signifies that effect is a continuous function of its cause" [2. P. 117].

The "striking analogy" of complete induction with the "usual processes of induction" lies in their function to be *tools* in order to structure different domains. These tools are suggested by experiences but are themselves inaccessible to experience. Poincaré's operative intuition is epistemic in character and signifies to take the decision that indefinite repetition, occasioned by experience, is a necessary tool for arithmetic.

Poincaré's inclusion of arithmetic under the book's title *Science and Hypothesis* may thus be justified in the following way: the *a priori* status of mathematical induction contrasts well with 'usual' hypotheses, but there is a striking analogy between the principle of mathematical induction and the status of a *natural hypothesis*: both are experimentally inaccessible rules. Poincaré is not sometimes intuitionist and sometimes formalist: he supports always the same position, adapted to the mathematical content.

In geometry, space is obtained by choosing the language of groups to serve as the tool of reasoning about representations of muscular sensations. Similarly to Carnap's "Aufbau", the starting point (guided by experience) is for Poincaré the definition of two two-place relations satisfying certain *minimal* empirical conditions: an *external change*  $a$  (with ' $x a y$ ' for ' $x$  changes in  $y$  *without* muscular sensation') and an *internal change*  $S$  (with ' $x S y$ ' for ' $x$  changes in  $y$  *accompanied by* muscular sensations'). Further, he proceeds to a conventional classification of external changes: among external changes some can be compensated by an internal change, others cannot. Such sensations changes are called **changes of position**. In this way, he obtains the following result: *modulo* an identity condition with respect to the compensation by internal changes, Poincaré defines the equivalence class of changes of position and calls it a *displacement*. Displacements form a group in the mathematical sense and it depends of the choice of its sub-groups whether the group corresponds to Euclidean or non Euclidean geometry.

Here is Poincaré's criterion for Euclidean geometry: "the existence of an invariant sub-group, of which all the displacements are interchangeable and which is formed out of all translations". In fact, the geometry of Euclid corresponds to the only group containing such an invariant group well founded by immediate experience [33. P. 21]. Consequently, the so-called *axiom* of Euclidean distance is not a definition by convention in a strictly linguistic sense, but the *result* of epistemic classifications. The defining group properties are clearly not empirical but they also cannot be the result of an "*a priori* reasoning". Indeed, they are suggested by experience and the displacements are rigorously defined by conventions.

Thus, experience plays a double role: it is at the same time the occasion for introducing the group, i. e. its *ratio cognoscendi*, and the coordinate definition in order to conceive of reality.

In summary, Poincaré's conventions could be called *disguised* definitions for two reasons: they are neither *proper* definitions, nor *proper* descriptions. They are distinguished by a mixed form, which finds a community of spirit in Goodman's reflective equilibrium principle.

We shall now comment on Poincaré's proof concept. According to Poincaré, mathematics requires intuition not only in the context of discovery but equally in the context of justification and, we already mentioned, that in arithmetic pure intuition is necessary to understand proofs.

From 1905 on, Poincaré thus refuses the twofold logicist's thesis:

– One can prove all mathematical truths with the help of purely “topic neutral”,

i. e. logical means;

– a logical proof is sufficient for our understanding.

This sounds very familiar because Wittgenstein defended later both argumentations of Poincaré:

What I want to find out is, whether it is true that once the principles of logic are admitted we can, I will not say discover, but demonstrate all mathematical truths without making a fresh appeal to intuition. To this question I formerly gave a negative answer. Must our answer be modified by recent works? [Non] [12. P. 465]	"The harmful thing about logical technique is that it makes us forget the special mathematical technique[...]
Logistics is no more than an auxiliary of intuition. [34. P. 145]	It is almost as if one tried to say that cabinet-making consisted in glueing" [35, V. P. 24]

While the first possibility of a purely logical formulation of proofs seems today widely accepted, the second point is much more interesting. Poincaré as Wittgenstein did not believe that formal logic, whatever its special form may be, could express the essential structure of a proof in view of its *understanding*:

When the logician has resolved each demonstration into a host of elementary operations, all of them correct, he will not yet be in possession of the whole reality; that indefinable something that constitutes the unity of the demonstration will still escape him completely [12. P. 448].	Everything that I say really amounts to this, that one can know a proof thoroughly and follow it step by step, and yet at the same time not <i>understand</i> what it was that was proved" [35, V. P. 25]
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In fact, together with a long philosophical tradition, mathematical practice shows that there exists an opposition between explanatory and non-explanatory proofs: the latter convince us but *do not explain*, give no “*deep*” reasons, do not give an *understanding*, etc., the former give not only the conviction *that* the result is true but show *why* it is true.

Poincaré's attempt to answer the challenge facing the difficulties of explanatory proofs consists in applying to the proof an understanding by the tool of a prag-

matic (Peircean) turn: the meaning of premises are connected with a semiotic analysis of their implications. From such a pragmatic point of view an explanatory proof may "exceed" a proof conceived as a modal sequence of propositions whose premises and conclusion are identified by means of models of a certain type. Some proofs are "better" than others because they include categories of mathematical *representation*. According to this account, reasoning links the content of the premises to the content of the conclusion by using intuitive 'seemings' and topic specific mathematical representations.

More precisely, Poincaré gives the following 'non-logical' criterion of a proof led by A:

1. A knows the guiding lines or, according to Poincaré's metaphor, its "architecture", i.e. the scaffolding of the stone arch which one gets rid, once the construction is completed [36. P. XIV].

2. There should be interdependency between the actual wood construction and the abstract curve that represents the upper contour.

3. The steps of a proof must themselves be arched. In other words, the reasoning that compose them, must themselves be singulars interdependent with the general they suggest.

For a pragmatist, *understanding* the truth of a mathematical sentence is a re-conceptualization of its "logical" proof, but not a meta-action that give "secondary standards" with respect to logic: rather it gives an answer to another why-question in addition to "why is the proof logical rigorous?". The explanatory content of the answer to this question is naturally not necessarily deeper with respect to the question asked than the logical reasoning was to the question of logical rigor.

Poincaré is interested in "why-questions" with respect to the "intuitive insight" into proofs and sees the non formal element as aesthetics: "It may appear surprising that sensibility should be introduced in connection with mathematical demonstrations, which, it would seem, can only interest the intellect. [...] It is a real aesthetic feeling that all true mathematicians recognize" [12. P. 396]. Indeed, how to make his metaphor of an esthetic architecture explicit?

In the spirit of Nelson Goodman, I argue for the thesis that the difference between the multiple ways of finding a cognitive evidence – by perception, by intuition of a first pre-linguistic level, or of a higher conceptual level, etc. – depends neither on the status the involved objects may have nor on the cognitive capacity used. Rather it is determined by the pragmatic use one makes of the semiotic system implied. The inseparability/interdependence between a singular and a general sign-perspective is a symptom of an intuitive use. The singular element *presents* a general.

In order that the intuitive presentation may be read as the *representation* of a general object, one has to introduce a symbolic aspect transforming the presentation to an *exemplification* in Goodman's sense. *A mathematical proof gives an "intuitive insight" if it contains intuitive language use, i.e. if "parts" of it can be interpreted as exemplifications of a general idea (schema). The insight increases in proportion to the intuitive proof stages and in proportion to the involved semantic density* (this replaces a strong purity criterion; [cf. 37]).

## V. Three characteristics of Poincaré's philosophy that are at the same time arguments in favor of Poincaré as a philosopher in analytic tradition

### 1. Conventions

As with Frege's "sense", Poincaré's "convention" is neither a description of empirical data nor of private events. In this respect, Poincaré survives Quine's criticism of logical empiricism by his conception of geometrical conventions as a kind of bicephalous selection of analytical but non-logical propositions, "guided" at the same time by experience.

However reluctantly, Poincaré prepared the way for later anti-inductivist philosophies of science. Although theories seem to be induced in slow stages from phenomena, the starting points are in reality a combination of "invented" phenomena and conceptual features (groups, *a priori* principles), put together in a pragmatic-conventional procedure which is irreducible to a combination of clearly distinguished parts of observation and logic.

### 2. Epistemic vs etioloical proof-attitude

Before Goodman's introduction of the cognitive turn in aesthetics [38], it was far from clear how to interpret Poincaré's statements in favor of the cognitive role of aesthetics in proofs. Signs of an aesthetic proof, i. e. a better explanatory proof, are semantic density, which accounts for its generativity, and syntactic repleteness in terms of the lack of superfluous semiotic marks (Poincaré introduced into mathematical reasoning Mach's principle of the economy of thought).

Today, to sustain an etioloical proof conception is not indicative for a non-analytical spirit.

### 3. The Non-Fregean Logic

Frege's use of topic-neutral versus domain-restricted general laws to distinguish between analytic and synthetic judgements is strongly reminiscent of Aristotle's prohibition of *metabasis eis allo genos*, that is, the shifting between various genera in *An Post* I:7: thus for instance, one must not use an arithmetical law to demonstrate a geometrical theorem.

Poincaré as well as Peirce apply the topic-neutral/domain-restricted distinction to different levels of complexity into mathematics: the use of geometry in Analysis is one of Poincaré's methodological masterpieces.

According to Hintikka, Poincaré's rejection of the Frege-type notation helped him to see the conceptual situation with respect to Richard's paradox more accurately (impredicativity).

Until Hintikka's switch from the traditional Frege-Russell first-order logic (in which not enough attention is paid to relations of quantifier dependence) to IF-Logic, logicians and mathematicians did not know how to implement Poincaré's insight.

Since the principle of induction is expressible in IF-Logic, to take creativity as a criterion for separating logic from mathematics is not longer possible. So, Poincaré's valuable argument against logic concerns only the level of understanding.

## Conclusion

Once the development of logic is revisited, Poincaré must be considered as one of the most prominent sources of the analytic tradition. His originality consists in

the fact that he works on the border between science centered and art centered philosophy: scientific questions have aesthetic solutions – in this sense he is an anti-wittgensteinian.

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#### **POINCARÉ AND THE ANALYTIC TRADITION**

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There is no substantive agreement about the philosophical views of analytic philosophers. Nevertheless, for much of them logical analysis is widely recognized to be important. It is true that Poincaré used no logical analysis but refused nevertheless the old metaphysics. Indeed, the analytic tradition of philosophy of science is perhaps better characterized by several overlapping similarities, which are the clincher for my main thesis: Analytic philosophy of science has one of its origins in the philosophical network in France around 1900 and, especially, in Poincaré. I mention the historical context but don't emphasize the stringency of Poincaré's argumentation. It's not a question of whether, in arguing as he did, he was always consistent (definitely, he was not), but to find out what lies behind the system he has built. Whereas the classical interpretation that Poincaré is Kantian in arithmetic and conventionalist in geometry leads to a potentially misleading understanding of his philosophical insights, such a misleading understanding can be overcome by classifying him as belonging to the analytic tradition. This is the reason why my main question is worth addressing not only from the historical point of view.