УДК 62-50
DOI: 10.17223/19988605/38/5

A.I. Rouban

THE SENSITIVITY FUNCTIONALS IN THE BOLTS'S PROBLEM FOR MULTIVARIATE DYNAMIC SYSTEMS DESCRIBED BY ORDINARY INTEGRAL EQUATIONS

The variation method is applied to calculation sensitivity functionals, which connect the first variation of quality functionals of systems operate (the Bolts's problem) with variations of variables and constant parameters, for the multidimensional nonlinear dynamic systems described by the generalized ordinary Volterra's second-kind integral equations.
Keywords: variation method; sensitivity functional; sensitivity coefficient; integral equation; conjugate equation.

The sensitivity functional (SF) connect the first variation of quality functional with variations of variable and constant parameters. Coefficients before variations of constant parameters name the sensitivity coefficients (SC). They are components of vector gradient from quality functional according to constant parameters.

The problem of calculation of SF and SC of dynamic systems is principal in the analysis and syntheses of control laws, identification, optimization [1–7]. The first-order sensitivity characteristics are mostly used. Later on we shall examine only SC and SF of the first-order.

Consider a vector output $y(t)$ of dynamic object model under continuous time $t \in [t_0, t^1]$, implicitly depending on vectors parameters $\tilde{\alpha}(t), \overline{\alpha}$ and functional I constructed on $y(t)$ under $t \in [t_0, t^1]$. The first variation δI of functional I and variations $\delta\tilde{\alpha}(t)$ are connected with each other with the help of a single-line functional – SF with respect to variable parameters $\tilde{\alpha}(t)$: $\delta_{\tilde{\alpha}(t)} I = \int_{t_0}^{t^1} V(t)\delta\tilde{\alpha}(t)dt$. SC with respect to constant parameters $\overline{\alpha}$ are called a gradient of I on $\overline{\alpha}$: $(dI/d\overline{\alpha})^T \equiv \nabla_{\overline{\alpha}} I$. SC are a coefficients of single-line relationship between the first variation of functional δI and the variations $\delta\overline{\alpha}$ of constant parameters $\overline{\alpha}$: $\delta_{\overline{\alpha}} I = (\nabla_{\overline{\alpha}} I)^T \delta\overline{\alpha} = (dI/d\overline{\alpha})\delta\overline{\alpha} \equiv \sum_{j=1}^{m} \frac{\partial I}{\partial \overline{\alpha}_j} \delta\overline{\alpha}_j$.

The direct method of SC calculation (by means of the differentiation of quality functional with respect to constant parameters) inevitably requires a solution of cumbersome sensitivity equations to sensitivity functions $W(t)$. $W(t)$ is the matrix of single-line relationship of the first variation of dynamic model output with parameter variations $\delta y(t) = W(t)\delta\overline{\alpha}$. For instance, for functional $I = \int_{t_0}^{t^1} f_0(y(t), \overline{\alpha}, t)dt$ we have following SC vector (row vector): $dI/d\overline{\alpha} = \int_{t_0}^{t^1} [(\partial f_0/\partial y)W(t) + \partial f_0/\partial\overline{\alpha}]dt$. For obtaining the matrix $W(t)$ it is necessary to decide bulky system equations – sensitivity equations. The j-th column of matrix $W(t)$ is made of the sensitivity functions $dy(t)/d\overline{\alpha}_j$ with respect to component $\overline{\alpha}_j$ of vector $\overline{\alpha}$. They satisfy a vector equation (if y is a vector) resulting from dynamic model (for y) by derivation [1–3] on a parameter $\overline{\alpha}_j$.

To variable parameters such a method is inapplicable because the sensitivity functions exist with respect to constant parameters.

For relatively simply classes of dynamic systems it is shown that in the SC calculation it is possible to get rid of deciding the bulky sensitivity equations due to the passage of deciding the conjugate equations – conjugate with respect to dynamic equations of object. Method of receipt of conjugate equations (it was offered in 1962) is cumbersome, because it is based on the analysis of sensitivity equations, and it does not get its development.

Variational method [4], ascending to Lagrange's, Hamilton's, Euler's memoirs, makes possible to simplify the process of determination of conjugate equations and formulas of account of SF and SC. On the basis of this method it is an extension of quality functional by means of inclusion into it object dynamic equations by means of Lagrange's multipliers and obtaining the first variation of extended functional on phase coordinates of object and on interesting parameters. Dynamic equations for Lagrange's multipliers are obtained due to set equal to a zero (in the first variation of extended functional) the functions before the first variations of phase coordinates. Given simplification first variation of extended functional brings at presence in the right part only parameter variations, i.e. it is got the SF. If all parameters are constant that the parameters variations are carried out from corresponding integrals and at the final result in obtained functional variation the coefficients before parameters variations are the required SC. Given method was used in [7–9] for dynamic systems described by ordinary continuous Volterra's second-kind integral and integro-differential equations. In this article the variational method of account of SC and of SF develops more general (on a comparison with papers [8, 9]) continuous many-dimensional non-linear dynamic systems circumscribed by the vectorial non-linear continuous ordinary Volterra's second-kind integral equations with variable and constant parameters. The more common quality functional (the Bolts's Problem) is used also.

1. Problem statement

We suppose that the dynamic object is described by system of non-linear continuous Volterra's ordinary integral equations (IE) of the second genus (more general than in the monography [7. P. 74]):

$$y(t) = r(\tilde{\alpha}(t),\overline{\alpha},t_0,t) + \int_{t_0}^{t} K(t, y(s), \tilde{\alpha}(s),\overline{\alpha},s)\, ds,\ t_0 \le t \le t^1,\ \ t_0 = t_0(\overline{\alpha}),\ t^1 = t^1(\overline{\alpha}). \tag{1}$$

Here: initial t_0 and final t^1 instants are known functions of constant parameters $\overline{\alpha}$. $\tilde{\alpha}(t), \overline{\alpha}$ are a vector-columns of interesting variable and constant parameters; y is a vector-column of phase coordinates; $r(\cdot)$, $K(\cdot)$ are known continuously differentiated limited vector-functions.

Variables $\eta(t)$ at each current moment of time t are connected with phase coordinates $y(t)$ by known transformation

$$\eta(t) = \eta(y(t),\tilde{\alpha}(t),\overline{\alpha},t),\quad t \in [t_0,t^1], \tag{2}$$

where $\eta(\cdot)$ – also continuous, continuously differentiable, limited (together with the first derivatives) vector-function. Equation (1.2) is often known as model of a measuring apparatus. The required parameters $\tilde{\alpha}(t),\overline{\alpha}$ are inserted also in it. A dimensionalities of vectors y and η can be various.

The quality of functioning of system it is characterised of functional

$$I = \int_{t_0}^{t^1} f_0(\eta(t),\tilde{\alpha}(t),\overline{\alpha},t)\,dt + I_1(\eta(t^1),\overline{\alpha},t^1), \tag{3}$$

depending on $\tilde{\alpha}(t)$ and $\overline{\alpha}$. The conditions for function $f_0(\cdot)$, $I_1(\cdot)$ are the same as for $K(\cdot)$, $r(\cdot)$. With use of a functional (1.3) the optimization problem (in the theory of optimal control) are named as the Bolts's problem. From it as the individual variants follow: Lagrange's problem (when there is only integral component) and Mayer's problem (when there is only second component – function from phase coordinates at a finishing point).

With the purpose of simplification of appropriate deductions with preservation of a generality in all transformations (1.1) – (1.3) there are two vectors of parameters $\tilde{\alpha}(t),\overline{\alpha}$. If in the equations (1.1)–(1.3) parameters are different then it is possible formally to unit them in two vectors $\tilde{\alpha}(t),\bar{\alpha}$, to use obtained outcomes and then to make appropriate simplifications, taking into account a structure of a vectors $\tilde{\alpha}(t),\bar{\alpha}$.

By obtaining of results the obvious designations:

$$r(t) \equiv r(\tilde{\alpha}(t),\bar{\alpha},t_0,t),\ K(t,s) \equiv K(t,y(s),\ \tilde{\alpha}(s),\bar{\alpha},s),\ \eta(t) \equiv \eta(y(t),\tilde{\alpha}(t),\bar{\alpha},t),$$

$$f_0(t) \equiv f_0(\eta(t),\tilde{\alpha}(t),\bar{\alpha},t),\ I_1(t^1) \equiv I_1(\eta(t^1),\bar{\alpha},t^1)$$

are used.

Is shown also that the variation method without basic modifications allows to receive SF $\delta I(\alpha) = \int_{t_0-\tau}^{t^1} V(t)\delta\tilde{\alpha}(t)dt + [dI(\alpha)/d\tilde{\alpha}(t^1)]\delta\tilde{\alpha}(t^1) + [dI(\alpha)/d\bar{\alpha}]\delta\bar{\alpha}$ in relation to variable and constant parameters.

2. Variational method for models (1)–(3)

Complement a quality functional (2) by restrictions-equalities (1) by means of Lagrange's multipliers $\gamma(t)$, $t \in [t_0,t^1]$, (column vectors) and get the extended functional

$$I = I(\alpha) + \int_{t_0}^{t^1} \gamma^T(t)[r(t) + \int_{t_0}^{t} K(t,s)\,ds - y(t)]\,dt\ , \tag{4}$$

which complies with $I(\alpha)$ when (1.1) is fulfilled. Take into account the form of functional I, change an order of integrating in double integral inside of triangular area (see fig. 1):

$$\left(\text{i.e.} \int_{t_0}^{t^1}\int_{t_0}^{t} A(t,s)\,ds\,dt = \int_{t_0}^{t^1}\int_{t}^{t^1} A(s,t)\,ds\,dt\right),$$

$$\int_{t_0}^{t^1} \gamma^T(t)\int_{t_0}^{t} K(t,s)\,ds\,dt = \int_{t_0}^{t^1}\int_{t}^{t^1} \gamma^T(s)K(s,t)\,ds\,dt\,, \tag{5}$$

and then extended functional (4) accepts a form:

$$I = I_1(t^1) + \int_{t_0}^{t^1} \{f_0(t) + \gamma^T(t)[r(t) - y(t)] + \int_{t}^{t^1} \gamma^T(s)K(s,t)\,ds\}\,dt\ . \tag{6}$$

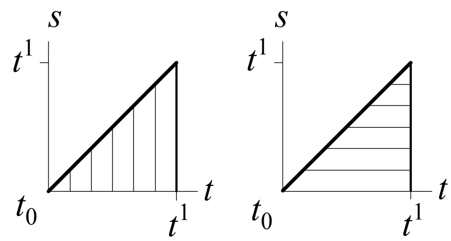

Fig. 1. Triangular area and order of an integration

Find the first variation for I with respect to $\delta y(t)$ and to $\delta\tilde{\alpha}(t)$ ($t \in [t_0,t^1)$), $\delta\tilde{\alpha}(t^1)$, $\delta\bar{\alpha}$ taking account: 1) dependence the right member of IE (1.1) on $y(t)$; 2) interconnection (3) between $\eta(t)$ and $y(t)$, $\tilde{\alpha}(t)$, $\bar{\alpha}$; 3) dependence t_0, t^1, $I_1(t^1)$ on $\bar{\alpha}$ [i.e. $t_0 = t_0(\bar{\alpha})$, $t^1 = t^1(\bar{\alpha})$, $I_1(t^1) \equiv I_1(\eta(t^1),\bar{\alpha},t^1)$]:

$$\delta I = \Phi(t^1)\,\delta y(t^1) + \int_{t_0}^{t^1}[\frac{\partial f_0(t)}{\partial\eta(t)}\frac{\partial\eta(t)}{\partial y(t)} + \int_{t}^{t^1}\gamma^T(s)\frac{\partial K(s,t)}{\partial y(t)}ds - \gamma^T(t)]\,\delta y(t)\,dt +$$

$$+\int_{t_0}^{t^1}[\frac{\partial f_0(t)}{\partial\eta(t)}\frac{\partial\eta(t)}{\partial\tilde{\alpha}(t)} + \frac{\partial f_0(t)}{\partial\tilde{\alpha}(t)} + \gamma^T(t)\frac{\partial r(t)}{\partial\tilde{\alpha}(t)} + \int_{t}^{t^1}\gamma^T(s)\frac{\partial K(s,t)}{\partial\tilde{\alpha}(t)}ds]\,\delta\tilde{\alpha}(t)\,dt +$$

$$+\frac{\partial I_1(t^1)}{\partial \eta(t^1)}\frac{\partial \eta(t^1)}{\partial \tilde{\alpha}(t^1)}\delta\tilde{\alpha}(t^1)+\left\{\frac{\partial I_1(t^1)}{\partial \eta(t^1)}\frac{\partial \eta(t^1)}{\partial \overline{\alpha}}+\frac{\partial I_1(t^1)}{\partial \overline{\alpha}}+\right.$$

$$+\int_{t_0}^{t^1}[\frac{\partial f_0(t)}{\partial \eta(t)}\frac{\partial \eta(t)}{\partial \overline{\alpha}}+\frac{\partial f_0(t)}{\partial \overline{\alpha}}+\gamma^T(t)\frac{\partial r(t)}{\partial \overline{\alpha}}+\int_t^{t^1}\gamma^T(s)\frac{\partial K(s,t)}{\partial \overline{\alpha}}ds]\,dt+$$

$$+\left[-f_0(t_0)+\int_{t_0}^{t^1}\gamma^T(t)(\frac{\partial r(t)}{\partial t_0}-K(t,t_0))dt\right]\frac{dt_0}{d\overline{\alpha}}+$$

$$\left.+\left[\frac{\partial I_1(t^1)}{\partial \eta(t^1)}\frac{\partial \eta(t^1)}{\partial t^1}+\frac{\partial I_1(t^1)}{\partial t^1}+f_0(t^1)\right]\frac{dt^1}{d\overline{\alpha}}\right\}\delta\overline{\alpha}\,, \tag{7}$$

here

$$\frac{\partial I_1(t^1)}{\partial \eta(t^1)}\frac{\partial \eta(t^1)}{\partial y(t^1)}\equiv \Phi(t^1).$$

Out of object equation (1) we calculate the first variation $\delta y(t^1)$ (variation, included in the first addend of (7))

$$\delta y(t^1)=\int_{t_0}^{t^1}\frac{\partial K(t^1,s)}{\partial y(s)}\delta y(s)\,ds+\int_{t_0}^{t^1}\frac{\partial K(t^1,s)}{\partial \tilde{\alpha}(s)}\delta\tilde{\alpha}(s)\,ds+$$

$$+\frac{\partial r(t^1)}{\partial \tilde{\alpha}(t^1)}\delta\tilde{\alpha}(t^1)+\left\{\frac{\partial r(t^1)}{\partial \overline{\alpha}}+\int_{t_0}^{t^1}\frac{\partial K(t^1,s)}{\partial \overline{\alpha}}ds+[\frac{\partial r(t^1)}{\partial t_0}-K(t^1,t_0)]\frac{dt_0}{d\overline{\alpha}}+\right.$$

$$\left.+[\frac{\partial r(t^1)}{\partial t^1}+K(t^1,t^1)+\int_{t_0}^{t^1}\frac{\partial K(t^1,s)}{\partial t^1}ds]\frac{dt^1}{d\overline{\alpha}}\right\}\delta\overline{\alpha}\,. \tag{8}$$

Then the first variation (7) obtains the following form:

$$\delta I=\delta_{y(t)}I+\delta_{\tilde{\alpha}(t)}I+\delta_{\alpha(t^1)}I+\delta_{\overline{\alpha}}I\,, \tag{9}$$

$$\delta_{y(t)}I=\int_{t_0}^{t^1}[\Phi(t^1)\frac{\partial K(t^1,t)}{\partial y(t)}+\frac{\partial f_0(t)}{\partial \eta(t)}\frac{\partial \eta(t)}{\partial y(t)}+\int_t^{t^1}\gamma^T(s)\frac{\partial K(s,t)}{\partial y(t)}ds-\gamma^T(t)]\delta y(t)\,dt\,, \tag{10}$$

$$\delta_{\tilde{\alpha}(t)}I=\int_{t_0}^{t^1}[\frac{\partial f_0(t)}{\partial \eta(t)}\frac{\partial \eta(t)}{\partial \tilde{\alpha}(t)}+\frac{\partial f_0(t)}{\partial \tilde{\alpha}(t)}+\gamma^T(t)\frac{\partial r(t)}{\partial \tilde{\alpha}(t)}+\Phi(t^1)\frac{\partial K(t^1,t)}{\partial \tilde{\alpha}(t)}+\int_t^{t^1}\gamma^T(s)\frac{\partial K(s,t)}{\partial \tilde{\alpha}(t)}ds]\delta\tilde{\alpha}(t)\,dt\,, \tag{11}$$

$$\delta_{\tilde{\alpha}(t^1)}I=[\frac{\partial I_1(t^1)}{\partial \eta(t^1)}\frac{\partial \eta(t^1)}{\partial \tilde{\alpha}(t^1)}+\Phi(t^1)\frac{\partial r(t^1)}{\partial \tilde{\alpha}(t^1)}]\delta\tilde{\alpha}(t^1)\,, \tag{12}$$

$$\delta_{\overline{\alpha}}I=\left\{\frac{\partial I_1(t^1)}{\partial \eta(t^1)}\frac{\partial \eta(t^1)}{\partial \overline{\alpha}}+\frac{\partial I_1(t^1)}{\partial \overline{\alpha}}+\Phi(t^1)[\frac{\partial r(t^1)}{\partial \overline{\alpha}}+\int_{t_0}^{t^1}\frac{\partial K(t^1,s)}{\partial \overline{\alpha}}ds]+\right.$$

$$+\int_{t_0}^{t^1}[\frac{\partial f_0(t)}{\partial \eta(t)}\frac{\partial \eta(t)}{\partial \overline{\alpha}}+\frac{\partial f_0(t)}{\partial \overline{\alpha}}+\gamma^T(t)\frac{\partial r(t)}{\partial \overline{\alpha}}+\int_t^{t^1}\gamma^T(s)\frac{\partial K(s,t)}{\partial \overline{\alpha}}ds]\,dt+$$

$$+\left[\Phi(t^1)[\frac{\partial r(t^1)}{\partial t_0}-K(t^1,t_0)]-f_0(t_0)+\int_{t_0}^{t^1}\gamma^T(t)[\frac{\partial r(t)}{\partial t_0}-K(t,t_0)]dt\right]\frac{dt_0}{d\overline{\alpha}}+$$

$$+\left[\Phi(t^1)[\frac{\partial r(t^1)}{\partial t^1}+K(t^1,t^1)+\int_{t_0}^{t^1}\frac{\partial K(t^1,s)}{\partial t^1}ds]+\right.$$

$$+\frac{\partial I_1(t^1)}{\partial\eta(t^1)}\frac{\partial\eta(t^1)}{\partial t^1}+\frac{\partial I_1(t^1)}{\partial t^1}+f_0(t^1)\Bigg]\frac{dt^1}{d\overline{\alpha}}\Bigg\}\delta\overline{\alpha}\,. \qquad (13)$$

In a variation (10) we equate with zero factors before variations of phase coordinates δy and discover: the conjugate equations for Lagrange's multipliers $\gamma(t)$

$$\gamma^T(t)=\Phi(t^1)\frac{\partial K(t^1,t)}{\partial y(t)}+\frac{\partial f_0(t)}{\partial\eta(t)}\frac{\partial\eta(t)}{\partial y(t)}+\int_t^{t^1}\gamma^T(s)\frac{\partial K(s,t)}{\partial y(t)}ds,\ t_0\le t\le t^1\,. \qquad (14)$$

These equations are decided in the opposite direction of time (from t^1).

In a result three components $\delta I=\delta_{\tilde{\alpha}(t)}I+\delta_{\tilde{\alpha}(t^1)}I+\delta_{\overline{\alpha}}I$ of the first variation of quality functional I in relation to variables $\tilde{\alpha}(t)$ and constant parameters $\tilde{\alpha}(t^1)$, $\overline{\alpha}$ are submitted accordingly by formulas (11), (12) and (13). This result is more common in relation to appropriate results of papers [7, 8]. Variables and constant parameters are present in integrated model of object, also at model of the measuring device and at generalized quality functional for system (the Bolts's Problem). An additional a dependence t_0,t^1 from $\overline{\alpha}$ are taken into account.

In a basis of calculation of sensitivity functionals the decision of the integrated equations of the object model in a forward direction of time and obtained integrated equations for Lagrange's multipliers in the opposite direction of time lays.

Example (The ordinary differential equations). Consider that the dynamic object is described by system of non-linear continuous differential equations with variable and constant parameters $\tilde{\alpha}(t)$, $\overline{\alpha}$:

$$\dot{y}(t)=f(y(t),\tilde{\alpha}(t),\overline{\alpha},t),\ \ t_0\le t\le t^1\,,\ y(t_0)=y_0(\overline{\alpha},t_0)\,. \qquad (15)$$

We transform model (15) in Volterra's second-kind integral equation (1)

$$y(t)=y_0(\overline{\alpha},t_0)+\int_{t_0}^{t}f(y(s),\tilde{\alpha}(s),\overline{\alpha},s)ds,\ t_0\le t\le t^1\,. \qquad (16)$$

Now

$$r(\cdot)=y_0(\overline{\alpha},t_0)\,,\ K(t,s)=f(y(s),\tilde{\alpha}(s),\overline{\alpha},s)\equiv f(s)\,.$$

We write the conjugate equations (2.11) for Lagrange's multipliers

$$\gamma^T(t)=\frac{\partial f_0(t)}{\partial\eta(t)}\frac{\partial\eta(t)}{\partial y(t)}+[\Phi(t^1)+\int_t^{t^1}\gamma^T(s)ds]\frac{\partial f(t)}{\partial y(t)},\ t_0\le t\le t^1\,,$$

and SF (11), (12), (13)

$$\delta I=\delta_{\tilde{\alpha}(t)}I+\delta_{\alpha(t^1)}I+\delta_{\overline{\alpha}}I\,,$$

$$\delta_{\tilde{\alpha}(t)}I=\int_{t_0}^{t^1}[\frac{\partial f_0(t)}{\partial\eta(t)}\frac{\partial\eta(t)}{\partial\tilde{\alpha}(t)}+\frac{\partial f_0(t)}{\partial\tilde{\alpha}(t)}+\Phi(t^1)\frac{\partial f(t)}{\partial\tilde{\alpha}(t)}+\int_t^{t^1}\gamma^T(s)\frac{\partial f(s)}{\partial\tilde{\alpha}(t)}ds]\delta\tilde{\alpha}(t)\,dt\,,$$

$$\delta_{\tilde{\alpha}(t^1)}I=\frac{\partial I_1(t^1)}{\partial\eta(t^1)}\frac{\partial\eta(t^1)}{\partial\tilde{\alpha}(t^1)}\delta\tilde{\alpha}(t^1)\,,$$

$$\delta_{\overline{\alpha}}I=\Bigg\{\frac{\partial I_1(t^1)}{\partial\eta(t^1)}\frac{\partial\eta(t^1)}{\partial\overline{\alpha}}+\frac{\partial I_1(t^1)}{\partial\overline{\alpha}}+\Phi(t^1)[\frac{\partial y_0(\overline{\alpha},t_0)}{\partial\overline{\alpha}}+\int_{t_0}^{t^1}\frac{\partial f(s)}{\partial\overline{\alpha}}ds]+$$

$$+\int_{t_0}^{t^1}[\frac{\partial f_0(t)}{\partial\eta(t)}\frac{\partial\eta(t)}{\partial\overline{\alpha}}+\frac{\partial f_0(t)}{\partial\overline{\alpha}}+\gamma^T(t)\frac{\partial y_0(\overline{\alpha},t_0)}{\partial\overline{\alpha}}+\int_t^{t^1}\gamma^T(s)ds]\frac{\partial f(t)}{\partial\overline{\alpha}}dt+$$

$$+\int_{t_0-\tau}^{t_0}\overline{\gamma}^T(t)\,dt\frac{\partial y_0(\overline{\alpha},t_0)}{\partial\overline{\alpha}}+\Bigg[\Phi(t^1)[\frac{\partial y_0(\overline{\alpha},t_0)}{\partial t_0}-f(t_0)]-$$

$$-f_0(t_0)+\int_{t_0}^{t^1}\gamma^T(t)dt[\frac{\partial y_0(\overline{\alpha},t_0)}{\partial t_0}-f(t_0)]\Bigg]\frac{dt_0}{d\overline{\alpha}}+$$

$$+\left[\Phi(t^1)f(t^1)+\frac{\partial I_1(t^1)}{\partial\eta(t^1)}\frac{\partial\eta(t^1)}{\partial t^1}+\frac{\partial I_1(t^1)}{\partial t^1}+f_0(t^1)\right]\frac{dt^1}{d\overline{\alpha}}\Bigg\}\delta\overline{\alpha}\,.$$

These results it is possible to represent in more customary (for differential equations) form. After change of variables:

$$\Phi(t^1)+\int_t^{t^1}\gamma^T(s)ds=\lambda^T(t),\ t_0\le t\le t^1\text{; ore } -\dot{\lambda}^T(t)=\gamma^T(t),\ t_0\le t\le t^1,\ \lambda^T(t^1)=\Phi(t^1)\,;$$

we obtain the conjugate equations in differential form

$$-\dot{\lambda}^T(t)=\frac{\partial f_0(t)}{\partial\eta(t)}\frac{\partial\eta(t)}{\partial y(t)}+\lambda^T(t)\frac{\partial f(t)}{\partial y(t)},\ \lambda^T(t^1)=\Phi(t^1),\ t_0\le t\le t^1,$$

and than SF have the form

$$\delta I=\delta_{\tilde{\alpha}(t)}I+\delta_{\alpha(t^1)}I+\delta_{\overline{\alpha}}I\,,$$

$$\delta_{\tilde{\alpha}(t)}I=\int_{t_0}^{t^1}[\frac{\partial f_0(t)}{\partial\eta(t)}\frac{\partial\eta(t)}{\partial\tilde{\alpha}(t)}+\frac{\partial f_0(t)}{\partial\tilde{\alpha}(t)}+\lambda^T(t)\frac{\partial f(t)}{\partial\tilde{\alpha}(t)}]\delta\tilde{\alpha}(t)\,dt\,,$$

$$\delta_{\tilde{\alpha}(t^1)}I=\frac{\partial I_1(t^1)}{\partial\eta(t^1)}\frac{\partial\eta(t^1)}{\partial\tilde{\alpha}(t^1)}\delta\tilde{\alpha}(t^1)\,,$$

$$\delta_{\overline{\alpha}}I=\Bigg\{\frac{\partial I_1(t^1)}{\partial\eta(t^1)}\frac{\partial\eta(t^1)}{\partial\overline{\alpha}}+\frac{\partial I_1(t^1)}{\partial\overline{\alpha}}+\lambda^T(t_0)\frac{\partial y_0(\overline{\alpha},t_0)}{\partial\overline{\alpha}}+$$

$$+\int_{t_0}^{t^1}[\frac{\partial f_0(t)}{\partial\eta(t)}\frac{\partial\eta(t)}{\partial\overline{\alpha}}+\frac{\partial f_0(t)}{\partial\overline{\alpha}}+\lambda^T(t)\frac{\partial f(t)}{\partial\overline{\alpha}}]dt+$$

$$+\left[\lambda^T(t_0)[\frac{\partial y_0(\overline{\alpha},t_0)}{\partial t_0}-f(t_0)]-f_0(t_0)\right]\frac{dt_0}{d\overline{\alpha}}+$$

$$+\left[\Phi(t^1)f(t^1)+\frac{\partial I_1(t^1)}{\partial\eta(t^1)}\frac{\partial\eta(t^1)}{\partial t^1}+\frac{\partial I_1(t^1)}{\partial t^1}+f_0(t^1)\right]\frac{dt^1}{d\overline{\alpha}}\Bigg\}\delta\overline{\alpha}\,.$$

Conclusion

The merit of variational method is applicability of its both for calculation of SF and SC. Besides the equations for Lagrange's multipliers remain without change.

Variables and constant parameters are present also at model of the measuring device and at generalized quality functional for system (the Bolts's Problem). In a basis of calculation of sensitivity functionals the decision of the integrated equations of model in a forward direction of time and obtained integrated equations for Lagrange's multipliers in the opposite direction of time lays.

Variation method of calculation of SF and SC allows a generalization on objects described by vectorial ordinary Volterra's second-kind integro-differential equations.

Integro-differential models structurally include separately integrated and differential models, and also 4 kinds of more simple integro-differential models which differ character of interaction of phase coordinates of integrated and differential parts. It is necessary to carry out transition from the integro-differential equation to corresponding integral eguation, to use results of this paper and in them to execute return to initial variables. For the objects described by simpler integro-differential models enough in the received connected equations and in SF and SC to turn into a zero a corresponding components.

Variation method of calculation of SF and SC allows a generalization on objects described by vectorial dynamic equations with delay time and different classes of discontinuous dynamic equations.

Results are applicable at design of high-precision systems and devices.

This paper continues research in [7–9].

REFERENCES

1. Ostrovskiy, G.M. & Volin, Yu.M. (1967) *Methods of optimization of chemical reactors*. Moscow: Khimiya. (In Russian).
2. Speedy, C.B., Brown, R.F. & Goodwin, G.C. (1973) *Control theory: identification and optimal control*. Tranlsated from English. Moscow: Mir. (In Russian).
3. Rosenvasser, E.N. & Yusupov, R.M. (eds) *Methods of sensitivity theory in automatic control*. Leningrad: Energiya. (In Russian)
4. Bryson, A.E. & Ho, Ju-Chi. (1972*) Applied optimal control*. Translated from English. Moscow: Mir. (In Russian).
5. Ruban, A.I. (1975) *Nonlinear dynamic object identification on the base of sensitivity algorithm*. Tomsk: Tomsk State University. (In Russian).
6. Rosenvasser, E.N. & Yusupov, R.M. (1981) *Sensitivity of control systems*. Moscow: Nauka. (In Russian).
7. Ruban, A.I. (1982) *Identification and sensitivity of complex systems*. Tomsk: Tomsk State University. (In Russian).
8. Ruban, A.I. (1996) Coefficients and functionals of sensitivity for multivariate systems described by integral equations. *Proceedings of Novosibirsk State Technical University*. 2(4). pp. 64–72. (In Russian).
9. Rouban, A.I. (1999) Coefficients and functionals of sensitivity for multivariate systems described by integral and integro-differetial equations. *Advances in Modeling & Analysis: Series A. Mathematical Problems; General Mathematical Modeling. France*: *A.M.S.E.* 35(1). pp. 25–34.

Rouban Anatoly Ivanovich. Dr. Science, prof. E-mail: ai-rouban@mail.ru
Siberial Federal University, Krasnoyarsk, Russian Federation

Поступила в редакцию 9 октября 2016 г.

Рубан Анатолий Иванович (Сибирский федеральный университет, г. Красноярск, Российская Федерация).

Функционалы чувствительности в задаче Больца для многомерных динамических систем, описываемых обыкновенными интегральными уравнениями.

Ключевые слова: вариационный метод; функционал чувствительности; обыкновенное интегральное уравнение; функционал качества работы системы; задача Больца; сопряженное уравнение.

DOI: 10.17223/19988605/38/5

Вариационный метод применен для расчета функционалов чувствительности, которые связывают первую вариацию функционалов качества работы систем с вариациями переменных и постоянных параметров, для многомерных нелинейных динамических систем, описываемых обобщенными обыкновенными интегральными уравнениями Вольтерра второго рода.