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MODELING THE EFFECT OF CENTERING OF A SPHERICAL HYDRODYNAMIC SUSPENSION

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Based on the methods of asymptotic integration in the case when the decentering force is orthogonal to the sensitivity axis, the fast centering of the spherical hydrodynamic suspension at large values of the oscillatory Reynolds number is shown. It is shown that, up to the constant factor, the asymptotics of the dependence of the relative eccentricity on the oscillatory Reynolds number is similar to the results obtained earlier for cylindrical hydrodynamic suspension.
Keywords: asymptotic integration; spherical hydrodynamic suspension.

High-load float gyroscopes are widely used elements of the control system of moving objects. In turn, spherical hydrodynamic suspension is a sensitive element of a number of float gyroscopes [1]. The stability on the mobile equilibrium curve of a weakly loaded spherical suspension at low oscillatory Reynolds numbers was studied earlier [1]. Cylindrical hydrodynamic suspension is quickly centered with increasing oscillatory Reynolds number [2]. Experimental data [1] confirm this effect for spherical hydrodynamic suspension. The influence of Coriolis forces of inertia on the dynamics of viscous incompressible fluid increases significantly with increasing oscillatory Reynolds numbers [3–9]. Nonlinear effects can destabilize the Couette flow in the layers between rotating spheres [10–14], but this is true when their angular velocities are significantly different. The aim of this work is to simulate the centering effect of a spherical hydrodynamic suspension based on the asymptotic integration of the reduced Navier-Stokes equations for the isothermal flow of a viscous incompressible fluid in a rotating support layer.

1. Mathematical model

We assume that the sensitivity axis coincides with the Oz axis of the $Oxyz$ coordinate system, the origin of which is in the center of mass of the inner sphere, and all the decentering forces lie in the Oxy plane. In disregard of some terms of the order of the square of the small relative thickness of the spherical layer (which is justified in [2]) and the small compressibility of the liquid, the equations of equilibrium of the suspension in dimensionless variables follow from [15] and take the form

$$\pi\gamma(\rho_2 / \rho - 1)(\mathbf{g} - \mathbf{a}_0) + \tfrac{3}{4}\int_0^{\pi} \sin\vartheta\, d\vartheta \int_0^{2\pi} d\varphi (\beta\sigma^{-1} h^{-1} (\partial v_\varphi / \partial x)\big|_{x=0} \mathbf{e}_\varphi - p\big|_{x=0} \mathbf{e}_r) = 0,$$

$$\tfrac{8}{3}\pi\beta\Omega + \int_0^{\pi} \sin^2\vartheta\, d\vartheta \int_0^{2\pi} (\partial v_\varphi / \partial x)\big|_{x=0} d\varphi / h = 0,\ \ h = \beta^{-1}[[(1+\beta)^2 + \beta^2((\mathbf{u}\cdot\mathbf{e}_r)^2 - \mathbf{u}^2)]^{1/2} - 1] - \mathbf{u}\cdot\mathbf{e}_r,$$

$$\mathbf{v} = \beta v_r \mathbf{e}_r + v_\vartheta \mathbf{e}_\vartheta + v_\varphi \mathbf{e}_\varphi,\ \ v_r = x(1+\beta h x)^{-1} \mathbf{v}\cdot\nabla^{(s)} h - (1+\beta h x)^{-2} \int_0^x \nabla^{(s)}\cdot(h(1+\beta h x)\mathbf{v})dx,$$

$$-\frac{1}{1+\beta hx}\frac{\partial\, p|_{x=0}}{\partial\vartheta}+\frac{1}{\sigma}\left(\frac{1}{h^2}\frac{\partial^2 v_\vartheta}{\partial x^2}+\frac{2\beta}{h}\frac{\partial v_\vartheta}{\partial x}\right)-\frac{\partial v_\vartheta}{\partial\varphi}+2v_\varphi\cos\vartheta+F_\vartheta[\mathbf{v}]=0,$$
$$-\frac{1}{1+\beta hx}\frac{1}{\sin\vartheta}\frac{\partial\, p|_{x=0}}{\partial\varphi}+\frac{1}{\sigma}\left(\frac{1}{h^2}\frac{\partial^2 v_\varphi}{\partial x^2}+\frac{2\beta}{h}\frac{\partial v_\varphi}{\partial x}\right)-\frac{\partial v_\varphi}{\partial\varphi}-2v_\vartheta\cos\vartheta-2\beta v_r\sin\vartheta+F_\varphi[\mathbf{v}]=0,$$
$$v_r\big|_{x=0}=0,\ \ v_\vartheta\big|_{x=0}=0,\ \ v_\varphi\big|_{x=0}=-\Omega\sin\vartheta,\ \ v_r\big|_{x=1}=\sin\vartheta(u_x\sin\varphi-u_y\cos\varphi),$$
$$v_\vartheta\big|_{x=1}=\beta\cos\vartheta(u_x\sin\varphi-u_y\cos\varphi),\ \ v_\varphi\big|_{x=1}=\beta(u_y\sin\varphi+u_x\cos\varphi),\quad W_0=-2v_\varphi\sin\vartheta+v_\vartheta^2+v_\varphi^2,\quad (1)$$
$$\nabla^{(s)}\cdot\boldsymbol{\Phi}^{(0)}[\mathbf{v}]=0,\ \ \boldsymbol{\Phi}^{(0)}[\mathbf{v}]=h\int_0^1(1+\beta hx)(v_\vartheta\mathbf{e}_\vartheta+v_\varphi\mathbf{e}_\varphi)dx-\tfrac{1}{2}(1+\beta hx)^2\boldsymbol{\Omega}_1^{(0)}\times\mathbf{u},$$
$$\mathbf{F}[\mathbf{v}]=\frac{1}{h}\left(\frac{\partial h}{\partial\varphi}x-v_r\right)\frac{\partial\mathbf{v}}{\partial x}-\frac{\beta}{1+\beta hx}\left[\nabla^{(s)}\left(h\int_0^x W_0dx\right)-xW_0\nabla^{(s)}h\right]-\beta v_r\mathbf{v}-(1-\beta hx)[\tfrac{1}{2}\nabla^{(s\xi)}(v_\vartheta^2+v_\varphi^2)]-$$
$$-(\mathbf{e}_r\times\mathbf{v})\nabla^{(s\xi)}\cdot(\mathbf{e}_r\times\mathbf{v})]+\beta^2[\nabla^{(s\xi)}(\nabla^{(s\xi)}\cdot\mathbf{v})-\mathbf{e}_r\times\nabla^{(s\xi)}(\nabla^{(s\xi)}\cdot(\mathbf{e}_r\times\mathbf{v}))]/\sigma.$$

Here $\mathbf{u}=(u_x,u_y,0)^{\mathrm{T}}$ and Ω are dimensionless displacement of the center of the inner sphere relative to the outer one and the difference of their angular velocities (sliding); h is the variable thickness of the support layer, $h(\pi-\vartheta,\varphi)=h(\vartheta,\varphi)$; $x=\xi/h\in[0,1]$ is a dimensionless deformed radial coordinate; ϑ, φ are the angular coordinates of a point on a spherical surface; $\mathbf{e}_r=(\sin\vartheta\cos\varphi,\sin\vartheta\sin\varphi,\cos\vartheta)^{\mathrm{T}}$, $\mathbf{e}_\vartheta=(\cos\vartheta\cos\varphi,\cos\vartheta\sin\varphi,-\sin\vartheta)^T$, $\mathbf{e}_\varphi=(-\sin\varphi,\cos\varphi,0)^{\mathrm{T}}$ are the unit vectors of a spherical coordinate system; ρ_2, ρ are the reduced density of the inner sphere and the density of the supporting layer, p is the dimensionless pressure, $p(\pi-\vartheta)=p(\vartheta)$; v_r, v_ϑ, v_φ are dimensionless radial and tangential components of fluid velocity, $v_{r,\varphi}(x,\pi-\vartheta,\varphi)=v_{r,\varphi}(x,\vartheta,\varphi)$, $v_\vartheta(x,\pi-\vartheta,\varphi)=-v_\vartheta(x,\vartheta,\varphi)$; $0\le\beta\ll 1$ is a small relative thickness of the supporting layer [2]; σ is the oscillatory Reynolds number [2]; γ characterizes the loading of the suspension; $\mathbf{g}=(0,-1,0)^{\mathrm{T}}$, $\mathbf{a}_0=(a_{0x},a_{0y},0)^{\mathrm{T}}$ are dimensionless vectors of free fall acceleration and overloads; $\boldsymbol{\Omega}_1^{(0)}=(0,0,1)^{\mathrm{T}}$. The spherical components of vector analysis operations, their analogues and the averaging operation act on the scalar field f and the vector field $\mathbf{F}=F_r\mathbf{e}_r+F_\vartheta\mathbf{e}_\vartheta+F_\varphi\mathbf{e}_\varphi$ as follows

$$\nabla^{(s)}f=\mathbf{e}_\vartheta\partial f/\partial\vartheta+\mathbf{e}_\varphi\sin^{-1}\vartheta\partial f/\partial\varphi,\ \ \nabla^{(s)}\cdot\mathbf{F}=\sin^{-1}\vartheta[\partial(F_\vartheta\sin\vartheta)/\partial\vartheta+\partial F_\varphi/\partial\varphi],$$
$$\nabla^{(s\xi)}f=\nabla^{(s)}f-xh^{-1}(\nabla^{(s)}h)\partial f/\partial x,\ \ \nabla^{(s\xi)}\cdot\mathbf{F}=\nabla^{(s)}\cdot\mathbf{F}-xh^{-1}\nabla^{(s)}h\cdot\partial\mathbf{F}/\partial x,\ \ \langle f\rangle=\int_0^1(1+\beta x)fdx.\quad (2)$$

2. Case of infinitely small thickness of the supporting layer

When $\beta=0$, the nonlinear boundary value problem (1) is simplified:

$$\langle f\rangle=\int_0^1 fdx;\ h=1-\mathbf{u}\cdot\mathbf{e}_r,\ \mathbf{v}=v_\vartheta\mathbf{e}_\vartheta+v_\varphi\mathbf{e}_\varphi,\ p=p(\vartheta,\varphi),\ \int_0^{2\pi}d\varphi\int_0^{\pi}\sin^2\vartheta\,(\partial v_\varphi/\partial x)\big|_{x=0}d\vartheta/h=0,$$
$$\pi\gamma(\rho_2/\rho-1)(\mathbf{g}-\mathbf{a}_0)-\tfrac{3}{4}\int_0^{2\pi}d\varphi\int_0^{\pi}\sin\vartheta d\vartheta p\mathbf{e}_r=0,\quad \partial v_r/\partial x=-h\nabla^{(s)}\cdot\mathbf{v}+x\nabla^{(s)}h\cdot\partial\mathbf{v}/\partial x,$$
$$-\frac{\partial p}{\partial\vartheta}+\frac{1}{\sigma h^2}\frac{\partial^2 v_\vartheta}{\partial x^2}-\frac{\partial v_\vartheta}{\partial\varphi}+2v_\varphi\cos\vartheta+F_\vartheta[\mathbf{v}]=0,\ \ -\frac{1}{\sin\vartheta}\frac{\partial p}{\partial\varphi}+\frac{1}{\sigma h^2}\frac{\partial^2 v_\varphi}{\partial x^2}-\frac{\partial v_\varphi}{\partial\varphi}-2v_\vartheta\cos\vartheta+\quad (3)$$
$$+F_\varphi[\mathbf{v}]=0;\ \nabla^{(s)}\cdot(h\langle\mathbf{v}\rangle-\tfrac{1}{2}\boldsymbol{\Omega}_1^{(0)}\times\mathbf{u})=0;\ v_r|_{x=0}=v_\vartheta|_{x=0}=v_\vartheta|_{x=1}=v_\varphi|_{x=1}=0,\ \ v_\varphi|_{x=0}=-\Omega\sin\vartheta,$$
$$\mathbf{F}[\mathbf{v}]=\frac{1}{h}\left(x\frac{\partial h}{\partial\varphi}-v_r\right)\frac{\partial\mathbf{v}}{\partial x}-\mathbf{e}_r\times\mathbf{v}\left[\nabla^{(s)}\cdot(\mathbf{e}_r\times\mathbf{v})-\frac{x}{h}\mathbf{e}_r\times\frac{\partial\mathbf{v}}{\partial x}\cdot\nabla^{(s)}h\right]-\nabla^{(s)}\frac{v_\vartheta^2+v_\varphi^2}{2}+\frac{\nabla^{(s)}h}{h}x\frac{\partial}{\partial x}\frac{v_\vartheta^2+v_\varphi^2}{2}.$$

The oscillatory Reynolds number σ is proportional to the first degree of the dimensional angular velocity of the outer sphere, and the parameter γ is inversely proportional to its second degree. Consequently, the solution of the nonlinear problem (3) is sought in the form of an asymptotic series

$$(\mathbf{u},\Omega,v_r,v_\vartheta,v_\varphi,p)=\gamma\sum_{k=0}^{\infty}\gamma^k(\mathbf{u}_k,\Omega_k,v_{r_k},v_{\vartheta_k},v_{\varphi_k},p_k),\ \gamma\ll 1,\quad \sigma=\underline{\underline{O}}(1)\,. \tag{4}$$

Here $\mathbf{u}_0,\Omega_0,v_{r_0},v_{\vartheta_0},v_{\varphi_0},p_0$ are the solution of a linear boundary value problem

$$\pi(\rho_2/\rho-1)(\mathbf{g}-\mathbf{a}_0)-\tfrac{3}{4}\int_0^{2\pi}d\varphi\int_0^{\pi}\sin\vartheta\, d\vartheta\, p\mathbf{e}_r=0,\quad \int_0^{2\pi}d\varphi\int_0^{\pi}\sin^2\vartheta\, d\vartheta\,(\partial v_\varphi/\partial x)\Big|_{x=0}=0\,, \tag{5}$$

$$\partial v_r/\partial x=-\nabla^{(s)}\cdot\mathbf{v},\ \ p=p(\vartheta,\varphi),\ \ \nabla^{(s)}\cdot\langle\mathbf{v}\rangle=\tfrac{1}{2}\sin\vartheta\left[(u_{0y}+iu_{0x})e^{i\varphi}+(u_{0y}-iu_{0x})e^{-i\varphi}\right],$$

$$-\frac{\partial p}{\partial\vartheta}+\frac{1}{\sigma}\frac{\partial^2 v_\vartheta}{\partial x^2}-\frac{\partial v_\vartheta}{\partial\varphi}+2v_\varphi\cos\vartheta=0,\quad -\frac{1}{\sin\vartheta}\frac{\partial p}{\partial\varphi}+\frac{1}{\sigma}\frac{\partial^2 v_\varphi}{\partial x^2}-\frac{\partial v_\varphi}{\partial\varphi}-2v_\vartheta\cos\vartheta=0, \tag{6}$$

$$v_r|_{x=0}=0,\ \ v_\vartheta|_{x=0}=0,\ \ v_\varphi|_{x=0}=-\Omega_0\sin\vartheta,\ \ v_\vartheta|_{x=1}=0,\ \ v_\varphi|_{x=1}=0.$$

The solution of the linear boundary value problem (6) has the form

$$\begin{gathered}p(\vartheta,\varphi)=\Omega_0p^{(0)}(\vartheta)+(u_{0y}+iu_{0x})e^{i\varphi}p^{(+)}(\vartheta)+(u_{0y}-iu_{0x})e^{-i\varphi}p^{(-)}(\vartheta),\ \ (\,)^{(-)}=\overline{(\,)^{(+)}},\\ v_{r,\vartheta,\varphi}(x,\vartheta,\varphi)=\Omega_0v^{(0)}_{r,\vartheta,\varphi}(x,\vartheta)+(u_{0y}+iu_{0x})e^{i\varphi}v^{(+)}_{r,\vartheta,\varphi}(x,\vartheta)+(u_{0y}-iu_{0x})e^{-i\varphi}v^{(-)}_{r,\vartheta,\varphi}(x,\vartheta).\end{gathered} \tag{7}$$

$$\begin{gathered}\frac{\partial v_r^{(0)}}{\partial x}=-\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}(v_\vartheta^{(0)}\sin\vartheta),\ \ \frac{\partial}{\partial\vartheta}(\sin\vartheta\left\langle v_\vartheta^{(0)}\right\rangle)=0,\ \ -\frac{\partial p^{(0)}}{\partial\vartheta}+\frac{1}{\sigma}\frac{\partial^2v_\vartheta^{(0)}}{\partial x^2}+2v_\varphi^{(0)}\cos\vartheta=0,\\ \sigma^{-1}\partial^2v_\varphi^{(0)}/\partial x^2-2v_\vartheta\cos\vartheta=0;\ \ v_r^{(0)}\Big|_{x=0}=v_\vartheta^{(0)}\Big|_{x=0}=0,\ \ v_\varphi^{(0)}\Big|_{x=0}=-\sin\vartheta,\ \ v_\vartheta^{(0)}\Big|_{x=1}=v_\varphi^{(0)}\Big|_{x=1}=0,\end{gathered} \tag{8}$$

$$\begin{gathered}\frac{\partial v_r^{(+)}}{\partial x}=-\frac{1}{\sin\vartheta}\left[\frac{\partial}{\partial\vartheta}(v_\vartheta^{(+)}\sin\vartheta)+iv_\varphi^{(+)}\right],\ \ \frac{\partial}{\partial\vartheta}\ \sin\vartheta\left\langle v_\vartheta^{(+)}\right\rangle\ +i\left\langle v_\varphi^{(+)}\right\rangle=\frac{1}{2}\sin^2\vartheta,\\ -\frac{\partial p^{(+)}}{\partial\vartheta}+\frac{1}{\sigma}\frac{\partial^2v_\vartheta^{(+)}}{\partial x^2}-iv_\vartheta^{(+)}+2v_\varphi^{(+)}\cos\vartheta=0,\ \ -\frac{i}{\sin\vartheta}p^{(+)}+\frac{1}{\sigma}\frac{\partial^2v_\varphi^{(+)}}{\partial x^2}-iv_\varphi^{(+)}-2v_\vartheta^{(+)}\cos\vartheta=0,\\ v_r^{(+)}\Big|_{x=0}=v_\vartheta^{(+)}\Big|_{x=0}=v_\varphi^{(+)}\Big|_{x=0}=0,\ \ v_\vartheta^{(+)}\Big|_{x=1}=v_\varphi^{(+)}\Big|_{x=1}=0.\end{gathered} \tag{9}$$

Subject to (7), the conditions (5) take the form

$$(u_{0y}+iu_{0x})\int_0^{\pi}\sin^2\vartheta\, p^{(+)}d\vartheta=\tfrac{2}{3}(\rho_2/\rho-1)(i(1+a_{0y})-a_{0x}),\ \ 2\pi\Omega_0\int_0^{\pi}\sin^2\vartheta\, d\vartheta\,(\partial v_\varphi^{(0)}/\partial x)\Big|_{x=0}=0. \tag{10}$$

In particular, solving the linear boundary value problem (8) and using (10), we find

$$(\partial v_\varphi^{(0)}/\partial x)\Big|_{x=0}=\sin\vartheta\,\mathrm{Re}\left[\sqrt{\tfrac{1}{2}i\sigma\cos\vartheta}\,\mathrm{cth}\sqrt{\tfrac{1}{2}i\sigma\cos\vartheta}\right],\quad \Omega_0=0\,. \tag{11}$$

and in this approximation, there is no sliding. The solution of (9) has the form

$$\begin{gathered}\left\langle v_\varphi^{(+)}\right\rangle=-\tfrac{1}{2}i\sin^2\vartheta+i\partial\ \sin\vartheta\left\langle v_\vartheta^{(+)}\right\rangle/\partial\vartheta\,,\ \ p^{(+)}=\tfrac{1}{4}\sin\vartheta[(\Theta_2^{-1}+\Theta_1^{-1})\left\langle v_\varphi^{(+)}\right\rangle+i(\Theta_2^{-1}-\Theta_1^{-1})\left\langle v_\vartheta^{(+)}\right\rangle],\\ v_\varphi^{(+)}=\tfrac{1}{2}(\left\langle v_\varphi^{(+)}\right\rangle-i\left\langle v_\vartheta^{(+)}\right\rangle)\Theta_1^{-1}(\vartheta)V_1(x,\cos\vartheta)+\tfrac{1}{2}(\left\langle v_\varphi^{(+)}\right\rangle+i\left\langle v_\vartheta^{(+)}\right\rangle)\Theta_2^{-1}(\vartheta)V_1(x,-\cos\vartheta),\\ v_\vartheta^{(+)}=\tfrac{1}{2}i(\left\langle v_\varphi^{(+)}\right\rangle-i\left\langle v_\vartheta^{(+)}\right\rangle)\Theta_1^{-1}(\vartheta)V_1(x,\cos\vartheta)-\tfrac{1}{2}i(\left\langle v_\varphi^{(+)}\right\rangle+i\left\langle v_\vartheta^{(+)}\right\rangle)\Theta_2^{-1}(\vartheta)V_1(x,-\cos\vartheta),\\ V_1(x,z)=\tfrac{1}{2}(1+2z)^{-1}\{[\mathrm{sh}((1-x)\sqrt{i\sigma(1+2z)})+\mathrm{sh}(x\sqrt{i\sigma(1+2z)})]/\,\mathrm{sh}\sqrt{i\sigma(1+2z)}-1\},\\ v_r^{(+)}=-\tfrac{1}{2}i\sin^{-1}\vartheta\{\partial[\Theta_1^{-1}(\vartheta)V_2(x,\cos\vartheta)(\left\langle v_\varphi^{(+)}\right\rangle-i\left\langle v_\vartheta^{(+)}\right\rangle)-\Theta_2^{-1}(\vartheta)V_2(x,-\cos\vartheta)(\left\langle v_\varphi^{(+)}\right\rangle+\\ +i\left\langle v_\vartheta^{(+)}\right\rangle)]/\partial\vartheta+\Theta_1^{-1}(\vartheta)V_2(x,\cos\vartheta)(\left\langle v_\varphi^{(+)}\right\rangle-i\left\langle v_\vartheta^{(+)}\right\rangle)+\Theta_2^{-1}(\vartheta)V_2(x,-\cos\vartheta)(\left\langle v_\varphi^{(+)}\right\rangle+i\left\langle v_\vartheta^{(+)}\right\rangle)\},\\ \Theta_1(\vartheta)=V_2(1,\cos\vartheta),\quad \Theta_2(\vartheta)=V_2(1,-\cos\vartheta),\ \ V_2(x,z)=\tfrac{1}{2}(1+2z)^{-1}\cdot\\ \cdot\{[\mathrm{ch}(x\sqrt{i\sigma(1+2z)})-\mathrm{ch}((1-x)\sqrt{i\sigma(1+2z)})-1+\mathrm{ch}\sqrt{i\sigma(1+2z)}]/(\sqrt{i\sigma(1+2z)}\,\mathrm{sh}\sqrt{i\sigma(1+2z)})-x\}.\end{gathered} \tag{12}$$

The function $\left\langle v_\vartheta^{(+)} \right\rangle$ is the solution of a linear ordinary differential equation

$$\frac{d}{d\vartheta}\left[\sin^3\vartheta\ \Theta_2^{-1}(\vartheta)+\Theta_1^{-1}(\vartheta)\ \frac{d}{d\vartheta}\left\langle v_\vartheta^{(+)}\right\rangle\right]=\Theta_3(\vartheta)-\Theta_4(\vartheta)\left\langle v_\vartheta^{(+)}\right\rangle, \quad (13)$$

where

$$\Theta_3(\vartheta)=\tfrac{1}{2}d[\sin^4\vartheta(\Theta_2^{-1}+\Theta_1^{-1})]/d\vartheta-\tfrac{1}{2}\cos\vartheta\sin^3\vartheta\ \Theta_2^{-1}+\Theta_1^{-1}\ -\tfrac{1}{2}\sin^3\vartheta\ \Theta_2^{-1}-\Theta_1^{-1}\ ,$$

$$\Theta_4(\vartheta)=\cos\vartheta\sin^2\vartheta d(\Theta_2^{-1}+\Theta_1^{-1})/d\vartheta+\sin^2\vartheta d(\Theta_2^{-1}-\Theta_1^{-1})/d\vartheta-2\sin^3\vartheta(\Theta_2^{-1}+\Theta_1^{-1}),$$

$$\Theta_3(\vartheta)=\underline{\underline{O}}(\sin^3\vartheta),\ \Theta_4(\vartheta)=\underline{\underline{O}}(\sin^3\vartheta),\ \vartheta\to 0,\ \vartheta\to\pi,$$

$$\Theta_4(\vartheta)=\underline{\underline{O}}(1/\sqrt{\sigma}),\ \sigma\to\infty,\ \vartheta\neq\pi/3,2\pi/3;\ \Theta_4(\pi/3)=\underline{\underline{O}}(1),\ \Theta_4(2\pi/3)=\underline{\underline{O}}(1),\ \sigma\to\infty.$$

Requirements for the absence of singularities of the solution of equation (13) in the poles of the sphere and anti-symmetry with respect to the equator $\vartheta=\pi/2$ lead to boundary conditions

$$\frac{d}{d\vartheta}\left\langle v_\vartheta^{(+)}\right\rangle\Big|_{\vartheta=0}=0,\quad \left\langle v_\vartheta^{(+)}\right\rangle\Big|_{\vartheta=\pi/2}=0. \quad (14)$$

When $\sigma\to\infty$, the asymptotic integration of the linear boundary value problem (13), (14) can be performed on the basis of the successive approximations method

$$\frac{d}{d\vartheta}\left[\sin^3\vartheta\ \Theta_2^{-1}+\Theta_1^{-1}\ \frac{d}{d\vartheta}\left\langle v_\vartheta^{(+)}\right\rangle_{k+1}\right]=$$

$$=\Theta_3-\Theta_4\left\langle v_\vartheta^{(+)}\right\rangle_k,\ \frac{d}{d\vartheta}\left\langle v_\vartheta^{(+)}\right\rangle_{k+1}\Big|_{\vartheta=0}=0,\left\langle v_\vartheta^{(+)}\right\rangle_{k+1}\Big|_{\vartheta=\pi/2}=0, \quad (15)$$

$$k=0,1,2,...,\ \left\langle v_\vartheta^{(+)}\right\rangle_0\equiv 0.$$

On the first iteration we assume $\Theta_2^{-1}+\Theta_1^{-1}\approx -4$, $\Theta_2^{-1}-\Theta_1^{-1}\approx 8\cos\vartheta$, and we find (see (10))

$$\left\langle v_\vartheta^{(+)}\right\rangle=-\tfrac{5}{8}\cos\vartheta,\ \int_0^\pi d\vartheta\sin^2\vartheta p^{(+)}=-\tfrac{3}{10}i,\ \sigma\to\infty, \quad (16)$$

$$u_{0x}=-\tfrac{20}{9}(\rho_2/\rho-1)a_{0x},\quad u_{0y}=-\tfrac{20}{9}(\rho_2/\rho-1)(1+a_{0y}). \quad (17)$$

On the second iteration, we find

$$\int_0^\pi d\vartheta\sin^2\vartheta p^{(+)}=-\tfrac{3}{10}i+C_p/\sqrt{\sigma},\ C_p=\tfrac{1971\sqrt{3}+4061}{17160\sqrt{2}}+i\tfrac{1971\sqrt{3}-4061}{17160\sqrt{2}},\ \sigma\to\infty. \quad (18)$$

The coefficients $\mathbf{u}_1,\Omega_1,v_{r_1},v_{\vartheta_1},v_{\varphi_1},p_1$ of asymptotic series (4) are the solution of the problem

$$\int_0^{2\pi}d\varphi\int_0^\pi\sin\vartheta d\vartheta p\mathbf{e}_r=0,\quad \int_0^{2\pi}d\varphi\int_0^\pi\sin^2\vartheta d\vartheta\,(\partial v_\varphi/\partial x+(\mathbf{u}_0\cdot\mathbf{e}_r)\partial v_{\varphi_0}/\partial x)\big|_{x=0}=0, \quad (19)$$

$$\partial v_r/\partial x=-\nabla^{(s)}\cdot\mathbf{v}+(\mathbf{u}_0\cdot\mathbf{e}_r)\nabla^{(s)}\cdot\mathbf{v}_0-x[(\mathbf{u}_0\cdot\mathbf{e}_\vartheta)\partial v_{\vartheta_0}/\partial x+(\mathbf{u}_0\cdot\mathbf{e}_\varphi)\partial v_{\varphi_0}/\partial x],$$

$$\mathbf{v}=v_\vartheta\mathbf{e}_\vartheta+v_\varphi\mathbf{e}_\varphi,\ \mathbf{v}_0=v_{\vartheta_0}\mathbf{e}_\vartheta+v_{\varphi_0}\mathbf{e}_\varphi,\ p=p(\vartheta,\varphi),\ \nabla^{(s)}\cdot\ \langle\mathbf{v}\rangle-(\mathbf{u}_0\cdot\mathbf{e}_r)\langle\mathbf{v}_0\rangle-\tfrac{1}{2}\boldsymbol{\Omega}_1^{(0)}\times\mathbf{u}_1\ =0,$$

$$-\frac{\partial p}{\partial\vartheta}+\frac{1}{\sigma}\left[\frac{\partial^2 v_\vartheta}{\partial x^2}+2(\mathbf{u}_0\cdot\mathbf{e}_r)\frac{\partial^2 v_{\vartheta_0}}{\partial x^2}\right]-\frac{\partial v_\vartheta}{\partial\varphi}+2v_\varphi\cos\vartheta+F_\vartheta^{(1)}[\mathbf{v}_0]=0,$$

$$-\frac{1}{\sin\vartheta}\frac{\partial p}{\partial\varphi}+\frac{1}{\sigma}\left[\frac{\partial^2 v_\varphi}{\partial x^2}+2(\mathbf{u}_0\cdot\mathbf{e}_r)\frac{\partial^2 v_{\varphi_0}}{\partial x^2}\right]-\frac{\partial v_\varphi}{\partial\varphi}-2v_\vartheta\cos\vartheta+F_\varphi^{(1)}[\mathbf{v}_0]=0, \quad (20)$$

$$v_r|_{x=0}=0,\ v_\vartheta|_{x=0}=0,\ v_\varphi|_{x=0}=-\Omega_1\sin\vartheta,\ v_\vartheta|_{x=1}=0,\ v_\varphi|_{x=1}=0,$$

$$\mathbf{F}^{(1)}[\mathbf{v}_0]=-(x\sin\vartheta(\mathbf{u}_0\cdot\mathbf{e}_\varphi)+v_{r_0})\partial\mathbf{v}_0/\partial x+(\mathbf{e}_r\times\mathbf{v}_0)\nabla^{(s)}\cdot(\mathbf{e}_r\times\mathbf{v}_0)-\tfrac{1}{2}\nabla^{(s)}(v_{\vartheta_0}^2+v_{\varphi_0}^2).$$

In equations (19) and (20)

$$v_{r_0,\vartheta_0,\varphi_0}(x,\vartheta,\varphi)=(u_{0y}+iu_{0x})e^{i\varphi}v_{r,\vartheta,\varphi}^{(+)}(x,\vartheta)+(u_{0y}-iu_{0x})e^{-i\varphi}v_{r,\vartheta,\varphi}^{(-)}(x,\vartheta),\ (\,)^{(-)}=\overline{(\,)^{(+)}}. \quad (21)$$

By virtue of linearity (20) we look for its solution in the form of

$$p(\vartheta,\varphi)=\Omega_1 p^{(0)}(\vartheta)+(u_{0x}^2+u_{0y}^2)p^{(1,0)}(\vartheta)+(\ldots)e^{i\varphi}+(\ldots)e^{-i\varphi}+(\ldots)e^{2i\varphi}+(\ldots)e^{-2i\varphi},$$
$$v_{r,\vartheta,\varphi}(x,\vartheta,\varphi)=\Omega_1 v_{r,\vartheta,\varphi}^{(0)}(x,\vartheta)+(u_{0x}^2+u_{0y}^2)v_{r,\vartheta,\varphi}^{(1,0)}(x,\vartheta)+(\ldots)e^{i\varphi}+(\ldots)e^{-i\varphi}+(\ldots)e^{2i\varphi}+(\ldots)e^{-2i\varphi}, \quad (22)$$

Moreover, in (22) the values $(\ldots)$ do not depend on Ω_1, and the functions $p^{(0)}(\vartheta)$ and $v_{r,\vartheta,\varphi}^{(0)}(x,\vartheta)$ are the solution (10). From (22), (21) and (19) follows

$$\Omega_1\int_0^\pi \sin^2\vartheta\, d\vartheta\,(\partial v_\varphi^{(0)}/\partial x)\Big|_{x=0}=(u_{0x}^2+u_{0y}^2)\int_0^\pi \sin^2\vartheta\, d\vartheta[\sin\vartheta\,\mathrm{Im}\,(\partial v_\varphi^{(+)}/\partial x)\Big|_{x=0}-(\partial v_\varphi^{(1,0)}/\partial x)\Big|_{x=0}]. \quad (23)$$

The functions $p^{(1,0)}(\vartheta)$ and $v_{r,\vartheta,\varphi}^{(1,0)}(x,\vartheta)$ are the solution of a linear boundary value problem

$$-dp^{(1,0)}\big/d\vartheta+\sigma^{-1}\,\partial^2 v_\vartheta^{(1,0)}\big/\partial x^2+2v_\varphi^{(1,0)}\cos\vartheta+f_\vartheta(x,\vartheta)=0,\ \sigma^{-1}\,\partial^2 v_\varphi^{(1,0)}\big/\partial x^2-2v_\vartheta^{(1,0)}\cos\vartheta+$$
$$+f_\varphi(x,\vartheta)=0;\ \ v_\vartheta^{(1,0)}\Big|_{x=0}=v_\varphi^{(1,0)}\Big|_{x=0}=v_\vartheta^{(1,0)}\Big|_{x=1}=v_\varphi^{(1,0)}\Big|_{x=1}=0,\ \left\langle v_\vartheta^{(1,0)}\right\rangle=\sin\vartheta\,\mathrm{Im}\left\langle v_\vartheta^{(+)}\right\rangle, \quad (24)$$
$$f_\vartheta(x,\vartheta)=-2\sigma^{-1}\sin\vartheta\,\mathrm{Im}\,\partial^2 v_\vartheta^{(+)}/\partial x^2-2\,\mathrm{Re}[(v_r^{(-)}+\tfrac{1}{2}x\sin\vartheta)\partial v_\vartheta^{(+)}/\partial x]-2\,\mathrm{Re}(v_\vartheta^{(-)}\partial v_\vartheta^{(+)}/\partial\vartheta)+$$
$$+2\sin^{-1}\vartheta\,\mathrm{Im}(v_\varphi^{(-)}v_\vartheta^{(+)})+2\left|v_\varphi^{(+)}\right|^2\mathrm{ctg}\,\vartheta,\ \ f_\varphi(x,\vartheta)=-2\,\mathrm{ctg}\,\vartheta\,\mathrm{Re}(v_\vartheta^{(-)}v_\varphi^{(+)})-$$
$$-2\sigma^{-1}\sin\vartheta\,\mathrm{Im}\,\partial^2 v_\varphi^{(+)}/\partial x^2-2\,\mathrm{Re}[(v_r^{(-)}+\tfrac{1}{2}x\sin\vartheta)\partial v_\varphi^{(+)}/\partial x]-2\,\mathrm{Re}(v_\vartheta^{(-)}\partial v_\varphi^{(+)}/\partial\vartheta).$$

As a result of the solution (24) we find

$$(\partial v_\varphi^{(1,0)}/\partial x)\Big|_{x=0}=\tfrac{1}{2}\sigma\int_0^1[(1+V_3(1-x,\vartheta)-V_3(x,\vartheta))f_\varphi(x,\vartheta)+(V_4(1-x,\vartheta)-$$
$$-V_4(x,\vartheta))f_\vartheta(x,\vartheta)+\sin 2\vartheta\,\mathrm{Im}\,v_\vartheta^{(+)}]dx,\ \ V_3(x,\vartheta)=\tfrac{1}{2}(V_5(x,\cos\vartheta)+V_5(x,-\cos\vartheta)),$$
$$V_4(x,\vartheta)=-\tfrac{1}{2}i(V_5(x,\cos\vartheta)-V_5(x,-\cos\vartheta)),\ \ V_5(x,z)=\mathrm{sh}(x\sqrt{2i\sigma z})/\mathrm{sh}\sqrt{2i\sigma z}.$$

The functions $V_{3,4}(1-x,\vartheta)-V_{3,4}(x,\vartheta)$ are odd in the variable x around the point $x=\frac{1}{2}$. We find

$$(\partial v_\varphi^{(1,0)}/\partial x)\Big|_{x=0}=-\tfrac{1}{2}\sigma\,\mathrm{Re}\,p^{(+)}+\sin\vartheta\,\mathrm{Im}\,(\partial v_\varphi^{(+)}/\partial x)\Big|_{x=0}-$$
$$-\sigma\sin^{-2}\vartheta\,\mathrm{Re}\,\partial[\sin^2\vartheta\int_0^1 dx v_\vartheta^{(-)}v_\varphi^{(+)}]/\partial\vartheta. \quad (25)$$

From (11), (23) and (25) follows

$$\tfrac{8}{21}\sqrt{\sigma}\Omega_1+\ldots=\tfrac{1}{2}\sigma(u_{0x}^2+u_{0y}^2)\int_0^\pi \sin^2\vartheta\, d\vartheta\,\mathrm{Re}\,p^{(+)},\ \ \sigma\to\infty. \quad (26)$$

From (18) and (26) we find

$$\Omega_1=C_\Omega(u_{0x}^2+u_{0y}^2),\ \ C_\Omega=\tfrac{21}{16}\mathrm{Re}\,C_p=\tfrac{7(1971\sqrt{3}+4061)}{91520\sqrt{2}}=0{,}404269,\ \ \sigma\to\infty. \quad (27)$$

Further, similarly to [2], we assume

$$\gamma=(\rho_2/\rho-1)^{-1}\chi\sigma^{-2},\ \ \chi=const=\underline{\underline{O}}(1),\ \ \sigma\to\infty. \quad (28)$$

From (4), (11), (17), (27) and (28) we find the asymptotic behavior of the relative eccentricity $\mathbf{u}$ and slip Ω as a function of the large oscillatory Reynolds numbers

$$\mathbf{u}=\mathbf{u}_0\sigma^{-2}+\bar{\bar{o}}(\sigma^{-2}),\ \ \mathbf{u}_0=(u_{0x},u_{0y},0)^T,\ \ \Omega=C_\Omega\sigma^{-4}+\bar{\bar{o}}(\sigma^{-4}),\ \ \sigma\to\infty,$$
$$u_{0x}=-\tfrac{20}{9}\chi a_{0x},\ \ u_{0y}=-\tfrac{20}{9}\chi(1+a_{0y}),\ \ C_\Omega=\tfrac{7(1971\sqrt{3}+4061)}{91520\sqrt{2}}=0{,}404269. \quad (29)$$

3. Effect of the finite thickness of the supporting layer

When $\beta\neq 0$ with asymptotic integration (1) at the first stage we assume that assumptions (4) are satisfied. The coefficients $\mathbf{u}_0,\Omega_0,v_{r_0},v_{\vartheta_0},v_{\varphi_0},p_0$ are the solution to the linear boundary value problem

$$\left(\frac{\rho_2}{\rho}-1\right)(\mathbf{g}-\mathbf{a}_0)+\frac{3}{4\pi}\int_0^{\pi}\sin\vartheta\,d\vartheta\int_0^{2\pi}d\varphi\left(\frac{\beta}{\sigma h}\frac{\partial v_\varphi}{\partial x}\bigg|_{x=0}\mathbf{e}_\varphi-p|_{x=0}\,\mathbf{e}_r\right)=0,\ \mathbf{v}=\beta v_r\mathbf{e}_r+v_\vartheta\mathbf{e}_\vartheta+v_\varphi\mathbf{e}_\varphi,$$

$$\tfrac{8}{3}\pi\beta\Omega_0+\int_0^{\pi}\sin\vartheta\,d\vartheta\int_0^{2\pi}(\partial v_\varphi/\partial x)\big|_{x=0}d\varphi/h=0,\ v_r=-(1+\beta x)^{-2}\int_0^x(1+\beta x)\nabla^{(s)}\cdot\mathbf{v}\,dx-$$

$$-\frac{1}{1+\beta x}\frac{\partial\,p|_{x=0}}{\partial\vartheta}+\frac{1}{\sigma}\left(\frac{\partial^2 v_\vartheta}{\partial x^2}+2\beta\frac{\partial v_\vartheta}{\partial x}\right)-\frac{\partial v_\vartheta}{\partial\varphi}+2v_\varphi\cos\vartheta+\frac{2\beta}{1+\beta x}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\int_0^x v_\varphi dx\right)+$$

$$+\frac{\beta^2}{\sigma}\left(\frac{\partial}{\partial\vartheta}\nabla^{(s)}\cdot\mathbf{v}+\frac{1}{\sin\vartheta}\frac{\partial}{\partial\varphi}\nabla^{(s)}\cdot(\mathbf{e}_r\times\mathbf{v})\right)=0,\quad-\frac{1}{1+\beta x}\frac{1}{\sin\vartheta}\frac{\partial\,p|_{x=0}}{\partial\varphi}+\frac{1}{\sigma}\left(\frac{\partial^2 v_\varphi}{\partial x^2}+2\beta\frac{\partial v_\varphi}{\partial x}\right)- \tag{30}$$

$$-\frac{\partial v_\varphi}{\partial\varphi}-2v_\vartheta\cos\vartheta-2\beta v_r\sin\vartheta+\frac{2\beta}{1+\beta x}\frac{\partial}{\partial\varphi}\int_0^x v_\varphi dx+\frac{\beta^2}{\sigma}\left(\frac{1}{\sin\vartheta}\frac{\partial}{\partial\varphi}\nabla^{(s)}\cdot\mathbf{v}-\frac{\partial}{\partial\vartheta}\nabla^{(s)}\cdot(\mathbf{e}_r\times\mathbf{v})\right)=0,$$

$$v_r|_{x=0}=0,v_\vartheta|_{x=0}=0,v_\varphi|_{x=0}=-\Omega_0\sin\vartheta,v_r|_{x=1}=\sin\vartheta(u_{0x}\sin\varphi-u_{0y}\cos\varphi),$$

$$v_\vartheta|_{x=1}=\beta\cos\vartheta(u_{0x}\sin\varphi-u_{0y}\cos\varphi),\quad v_\varphi|_{x=1}=\beta(u_{0y}\sin\varphi+u_{0x}\cos\varphi),$$

$$\nabla^{(s)}\cdot\left\langle v_\vartheta\mathbf{e}_\vartheta+v_\varphi\mathbf{e}_\varphi\right\rangle=-(1+\beta)^2\sin\vartheta(u_{0x}\sin\varphi-u_{0y}\cos\varphi).$$

Solution of (30) is sought in the form (see also (11))

$$\Omega_0=0,\ p|_{x=0}=(u_{0y}+iu_{0x})e^{i\varphi}p^{(+)}(\vartheta)+(u_{0y}-iu_{0x})e^{-i\varphi}p^{(-)}(\vartheta),$$
$$v_{r,\vartheta,\varphi}(x,\vartheta,\varphi)=(u_{0y}+iu_{0x})e^{i\varphi}v^{(+)}_{r,\vartheta,\varphi}(x,\vartheta)+(u_{0y}-iu_{0x})e^{-i\varphi}v^{(-)}_{r,\vartheta,\varphi}(x,\vartheta),\ (\,)^{(-)}=\overline{(\,)^{(+)}}. \tag{31}$$

where do the boundary value problem with respect to $p^{(+)}(\vartheta)$, $v^{(+)}_{r,\vartheta,\varphi}(x,\vartheta)$ come from

$$(u_{0y}+iu_{0x})\int_0^{\pi}d\vartheta\left(\sin^2\vartheta\,p^{(+)}+\frac{i\beta}{\sigma}\frac{\partial v_\varphi^{(+)}}{\partial x}\bigg|_{x=0}\right)=\frac{2}{3}\left(\frac{\rho_2}{\rho}-1\right)(-a_{0x}+i(1+a_{0y})), \tag{32}$$

$$v_r^{(+)}=-(1+\beta x)^{-2}\sin^{-1}\vartheta[\partial(\sin\vartheta\int_0^x(1+\beta x)v_\vartheta^{(+)}dx)/\partial\vartheta+i\int_0^x(1+\beta x)v_\varphi^{(+)}dx],\ 2v_\varphi^{(+)}\cos\vartheta-iv_\vartheta^{(+)}-$$

$$-\frac{1}{1+\beta x}\frac{dp^{(+)}}{d\vartheta}+\frac{1}{\sigma}\left(\frac{\partial^2 v_\vartheta^{(+)}}{\partial x^2}+2\beta\frac{\partial v_\vartheta^{(+)}}{\partial x}\right)+\frac{2\beta}{1+\beta x}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\int_0^x v_\varphi^{(+)}dx\right)+\frac{\beta^2}{\sigma}\left(\frac{\partial^2 v_\vartheta^{(+)}}{\partial\vartheta^2}+\mathrm{ctg}\,\vartheta\frac{\partial v_\vartheta^{(+)}}{\partial\vartheta}-\right.$$

$$\left.-2\frac{v_\vartheta^{(+)}}{\sin^2\vartheta}-2\frac{\cos\vartheta}{\sin^2\vartheta}iv_\varphi^{(+)}\right)=0,\quad-\frac{1}{1+\beta x}\frac{i}{\sin\vartheta}p^{(+)}+\frac{1}{\sigma}\left(\frac{\partial^2 v_\vartheta^{(+)}}{\partial x^2}+2\beta\frac{\partial v_\vartheta^{(+)}}{\partial x}\right)+\frac{2\beta}{1+\beta x}i\int_0^x v_\varphi^{(+)}dx+ \tag{33}$$

$$+\frac{\beta^2}{\sigma}\left(\frac{\partial^2 v_\varphi^{(+)}}{\partial\vartheta^2}+\mathrm{ctg}\,\vartheta\frac{\partial v_\varphi^{(+)}}{\partial\vartheta}-2\frac{v_\varphi^{(+)}}{\sin^2\vartheta}+2\frac{\cos\vartheta}{\sin^2\vartheta}iv_\vartheta^{(+)}\right)-iv_\varphi^{(+)}-2v_\vartheta^{(+)}\cos\vartheta-2\beta v_r^{(+)}\sin\vartheta=0,$$

$$v_\vartheta^{(+)}\big|_{x=0}=0,\ v_\varphi^{(+)}\big|_{x=0}=-\sin\vartheta,\ v_r^{(+)}\big|_{x=0}=0,\ v_\vartheta^{(+)}\big|_{x=1}=-\tfrac{1}{2}\beta\cos\vartheta,\ v_\varphi^{(+)}\big|_{x=1}=-\tfrac{1}{2}i\beta,$$

$$v_r^{(+)}\big|_{x=1}=-\tfrac{1}{2}\sin\vartheta;\ \sin^{-1}\vartheta[\partial(\sin\vartheta\left\langle v_\vartheta^{(+)}\right\rangle)/\partial\vartheta+i\left\langle v_\varphi^{(+)}\right\rangle]=\tfrac{1}{2}(1+\beta)^2\sin\vartheta.$$

We assume that (28) is true. When $\sigma\gg 1$, we look for a solution of (33) in the form

$$v^{(+)}_{r,\vartheta,\varphi}=v^{(+,e)}_{r,\vartheta,\varphi}(x,\vartheta)+\bar{\bar{o}}(1)\text{ in the area away from borders }x=0\text{ and }x=1;$$
$$v^{(+)}_{\vartheta,\varphi}=v^{(+,i_0)}_{\vartheta,\varphi}(\eta,\vartheta)+\bar{\bar{o}}(1),\ v_r^{(+)}=\bar{\bar{o}}(1),\ \eta=x\sqrt{\sigma}\text{ near }x=0; \tag{34}$$
$$v^{(+)}_{\vartheta,\varphi}=v^{(+,i_1)}_{\vartheta,\varphi}(\zeta,\vartheta)+\bar{\bar{o}}(1),\ v_r^{(+)}=-\tfrac{1}{2}\sin\vartheta+\bar{\bar{o}}(1),\ \zeta=(1-x)\sqrt{\sigma}\text{ near }x=1.$$

Boundary conditions are set for functions $v^{(+,i_0)}_{\vartheta,\varphi}$ and $v^{(+,i_1)}_{\vartheta,\varphi}$, boundary conditions for $v_r^{(+,e)}$ are obtained from the conditions of matching asymptotic representations (34). Similarly (12), matching is possible everywhere except for the narrow neighborhoods of the support layer corresponding to $\vartheta=\pi/3$ and $\vartheta=2\pi/3$. These linear boundary value problems follow from (34) and (33)

$$v_r^{(+,e)} = -(1+\beta x)^{-2}\sin^{-1}\vartheta[\partial(\sin\vartheta\int_0^x(1+\beta x)v_\vartheta^{(+,e)}dx)/\partial\vartheta + i\int_0^x(1+\beta x)v_\varphi^{(+,e)}dx],$$

$$-(1+\beta x)^{-1}\sin^{-1}\vartheta\, ip^{(+)} - iv_\varphi^{(+,e)} - 2v_\vartheta^{(+,e)}\cos\vartheta - 2\beta v_r^{(+,e)}\sin\vartheta + 2\beta(1+\beta x)^{-1} i\int_0^x v_\varphi^{(+,e)}dx = 0,$$

$$-(1+\beta x)^{-1}dp^{(+)}/d\vartheta - iv_\vartheta^{(+,e)} + 2v_\varphi^{(+,e)}\cos\vartheta + 2\beta(1+\beta x)^{-1}\partial(\sin\vartheta\int_0^x v_\varphi^{(+)}dx)/\partial\vartheta = 0, \quad (35)$$

$$v_r^{(+,e)}\Big|_{x=0} = 0,\ \ v_r^{(+,e)}\Big|_{x=1} = -\tfrac{1}{2}\sin\vartheta;\ \partial(\sin\vartheta\left\langle v_\vartheta^{(+,e)}\right\rangle)/\partial\vartheta + i\left\langle v_\varphi^{(+,e)}\right\rangle = \tfrac{1}{2}(1+\beta)^2\sin^2\vartheta.$$

$$v_\vartheta^{(+,i_0)} = v_\vartheta^{(+,e)}\Big|_{x=0} + w_\vartheta,\ \ v_\varphi^{(+,i_0)} = v_\varphi^{(+,e)}\Big|_{x=0} + w_\varphi,$$

$$\partial^2 w_\vartheta/\partial\eta^2 - iw_\vartheta + 2w_\varphi\cos\vartheta = 0,\ \ \partial^2 w_\varphi/\partial\eta^2 - iw_\varphi - 2w_\vartheta\cos\vartheta = 0, \quad (36)$$

$$w_\vartheta|_{\eta=0} = -v_\vartheta^{(+,e)}\Big|_{x=0},\ \ w_\varphi|_{x=0} = -v_\varphi^{(+,e)}\Big|_{x=0},\ \ w_\vartheta|_{\eta=\infty} = w_\varphi|_{\eta=\infty} = 0.$$

The solution of the (36) has the form

$$v_\varphi^{(+,i_0)} = \tfrac{1}{2}(i\,v_\vartheta^{(+,e)}\Big|_{x=0} - v_\varphi^{(+,e)}\Big|_{x=0})e^{-\eta\sqrt{i(1+2\cos\vartheta)}} - \tfrac{1}{2}(i\,v_\vartheta^{(+,e)}\Big|_{x=0} + v_\varphi^{(+,e)}\Big|_{x=0})e^{-\eta\sqrt{i(1-2\cos\vartheta)}} - v_\varphi^{(+,e)}\Big|_{x=0},$$

$$v_\varphi^{(+,i_0)} = \tfrac{1}{2}(i\,v_\varphi^{(+,e)}\Big|_{x=0} - v_\vartheta^{(+,e)}\Big|_{x=0})e^{-\eta\sqrt{i(1-2\cos\vartheta)}} - \tfrac{1}{2}(v_\vartheta^{(+,e)}\Big|_{x=0} + i\,v_\varphi^{(+,e)}\Big|_{x=0})e^{-\eta\sqrt{i(1+2\cos\vartheta)}} - v_\vartheta^{(+,e)}\Big|_{x=0}. \quad (37)$$

From (37) and (32) follows

$$(u_{0y} + iu_{0x})\int_0^\pi d\vartheta\sin^2\vartheta\, p^{(+)} = \tfrac{2}{3}(\rho_2/\rho - 1)(-a_{0x} + i(1+a_{0y})) + \underline{\underline{O}}(1/\sqrt{\sigma}),\ \ \sigma \gg 1. \quad (38)$$

Since $\beta \ll 1$, a further solution (35) is sufficient to hold up to $\underline{\underline{O}}(\beta^2)$. Up to small order $\underline{\underline{O}}(\beta)$, $v_\vartheta^{(+,e)}$ and $v_\varphi^{(+,e)}$ are independent of x, and $v_r^{(+,e)}$ is a linear function of x. That is

$$\left\langle\beta v_r^{(+,e)}\right\rangle = -\tfrac{1}{4}\beta\sin\vartheta + \underline{\underline{O}}(\beta^2),\ \ \left\langle\beta(1+\beta x)^{-1}\int_0^x v_\varphi^{(+,e)}dx\right\rangle = \tfrac{1}{2}\beta\left\langle v_\varphi^{(+,e)}\right\rangle + \underline{\underline{O}}(\beta^2). \quad (39)$$

From (39) and (35) we find

$$-\frac{dp^{(+)}}{d\vartheta} - i\left\langle v_\vartheta^{(+,e)}\right\rangle + 2\left\langle v_\varphi^{(+,e)}\right\rangle\cos\vartheta + \beta\frac{d}{d\vartheta}\ \sin\vartheta\left\langle v_\varphi^{(+,e)}\right\rangle\ + \underline{\underline{O}}(\beta^2) = 0,$$

$$-i\sin^{-1}\vartheta\, p^{(+)} - i(1-\beta)\left\langle v_\varphi^{(+,e)}\right\rangle - 2\left\langle v_\vartheta^{(+,e)}\right\rangle\cos\vartheta + \underline{\underline{O}}(\beta^2) = -\tfrac{1}{4}\beta\sin^2\vartheta,$$

$$\frac{d}{d\vartheta}\ \sin\vartheta\left\langle v_\vartheta^{(+,e)}\right\rangle\ + i\left\langle v_\varphi^{(+,e)}\right\rangle = \frac{1}{2}(1+\beta)^2\sin^2\vartheta,$$

whence it follows that

$$\left\langle v_\varphi^{(+,e)}\right\rangle = -i\left[\frac{1}{2}(1+\beta)^2\sin^2\vartheta - \frac{d}{d\vartheta}\ \sin\vartheta\left\langle v_\vartheta^{(+,e)}\right\rangle\ \right],$$

$$p^{(+)} = \frac{i}{2}\left(1+\frac{\beta}{2}\right)\sin^3\vartheta + 2i\cos\vartheta\sin\vartheta\left\langle v_\vartheta^{(+,e)}\right\rangle - i(1-\beta)\sin\vartheta\frac{d}{d\vartheta}\ \sin\vartheta\left\langle v_\vartheta^{(+,e)}\right\rangle\ , \quad (40)$$

$$\frac{d}{d\vartheta}\left(\sin^3\vartheta\frac{d}{d\vartheta}\left\langle v_\vartheta^{(+,e)}\right\rangle\right) = \left(\frac{5}{2} + \frac{17}{4}\beta\right)\sin^3\vartheta\cos\vartheta. \quad (41)$$

Requirements for the absence of singularities at $\vartheta = 0$ and asymmetry with respect to $\vartheta = \pi/2$ lead to boundary conditions

$$\frac{d}{d\vartheta}\left\langle v_\vartheta^{(+,e)}\right\rangle\Big|_{\vartheta=0} = 0,\ \ \left\langle v_\vartheta^{(+,e)}\right\rangle\Big|_{\vartheta=\pi/2} = 0. \quad (42)$$

The solution of (41), (42) has the form

$$\left\langle v_\vartheta^{(+,e)}\right\rangle = -(\tfrac{5}{8} + \tfrac{17}{16}\beta)\cos\vartheta. \quad (43)$$

From (43), (40), (38), (4) we find

$$\int_0^\pi \sin^2\vartheta\, d\vartheta\, p^{(+)} = -i(\tfrac{3}{10} + \tfrac{11}{30}\beta),$$

$$\mathbf{u} = \mathbf{u}_0 \sigma^{-2} + \bar{\bar{o}}(\sigma^{-2}),\ \sigma \to \infty,\ \mathbf{u}_0 = (u_{0x}, u_{0y}, 0)^{\mathrm{T}},$$
$$u_{0x} = -\frac{20}{9}\left(1 - \frac{11}{9}\beta\right)\chi a_{0x},\ u_{0y} = -\frac{20}{9}\left(1 - \frac{11}{9}\beta\right)\chi (1 + a_{0y}).$$

Taking into account (4) and (29) we can assume

$$\Omega = \underline{\underline{O}}(\beta \sigma^{-7/2} + \sigma^{-4}),\ \sigma \to \infty.$$

Conclusion

Based on the methods of asymptotic integration in the case when the decentering force is orthogonal to the sensitivity axis, the fast centering of the spherical hydrodynamic suspension at large values of the oscillatory Reynolds number is shown. It is shown that, up to the constant factor, the asymptotics of the dependence of the relative eccentricity on the oscillatory Reynolds number is similar to the results obtained earlier for cylindrical hydrodynamic suspension. However, the asymptotics of a similar dependence for the dimensionless sliding of the inner sphere is characterized by an order of magnitude smaller.

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Андрейченко Д.К., Андрейченко К.П., Мельничук Д.В. МОДЕЛИРОВАНИЕ ЭФФЕКТА ЦЕНТРИРОВАНИЯ СФЕРИЧЕСКОГО ГИДРОДИНАМИЧЕСКОГО ПОДВЕСА. *Вестник Томского государственного университета. Управление, вычислительная техника и информатика.* 2020. № 52. С. 13–21

Высокоперегрузочные поплавковые гироскопы являются широко используемыми элементами систем управления подвижными объектами. В свою очередь, сферический гидродинамический подвес является широко используемым чувствительным элементом ряда поплавковых гироскопов. В случае, когда децентрирующая сила ортогональна оси чувствительности прибора, на основе методов асимптотического интегрирования показано быстрое центрирование подвеса при больших значениях колебательного числа Рейнольдса. Математическое моделирование изотермического течения вязкой несжимаемой жидкости во вращающемся поддерживающем слое выполнено на основе укороченных уравнений Навье–Стокса. Показано, что с точностью до постоянного сомножителя асимптотика зависимости относительного эксцентриситета от колебательного числа Рейнольдса аналогична результатам, полученным ранее для цилиндрического гидродинамического подвеса. Однако асимптотика аналогичной зависимости для безразмерного скольжения внутренней сферы характеризуется на порядок меньшей величиной.

Ключевые слова: асимптотическое интегрирование; сферический гидродинамический подвес.

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