ВЕСТНИК ТОМСКОГО ГОСУДАРСТВЕННОГО УНИВЕРСИТЕТА

2021 Управление, вычислительная техника и информатика № 57

UDK 62–50
DOI: 10.17223/19988605/57/10

A.I. Rouban

THE SENSITIVITY COEFFICIENTS FOR DYNAMIC SYSTEMS DESCRIBED BY DIFFERENCE EQUATIONS WITH THE DISTRIBUTED MEMORY ON PHASE COORDINATES AND VARIABLE PARAMETERS

The variational method of calculation of sensitivity coefficients connecting first variation of quality functional with variations of variable and constant parameters for multivariate non-linear dynamic systems described by difference equations with the distributed memory on phase coordinates and variable parameters is developed. Sensitivity coefficients are components of sensitivity functional and they are before variations of variable and constant parameters. The base of calculation are the decision of object equations in the forward direction of discrete time and corresponding difference conjugate equations for Lagrange's multipliers in the opposite direction of discrete time.

Keywords: variational method; sensitivity coefficient; difference equation; conjugate equation; Lagrange's multiplier.

For dynamic systems the problem of calculation of sensitivity coefficients (SC) is central at the analysis and syntheses of control laws, optimization, identification, stability [1–16]. The first-order sensitivity characteristics mostly are used. Later on we shall examine only SC of the first-order.

The sensitivity functional connects the first variation of quality functional with variations of variable and constant parameters and the SC are components of vector gradient from quality functional according to parameters.

Consider a vector output $y(t)$ of dynamic object model under discrete time $t \in [0,1,...,N+1]$ implicitly depending on vector constant parameters α and additive functional $I(\alpha)$ constructed on a basis of $y(t)$ under $t \in [0,1,...,N+1]$ and on a basis of parameters α :

$$I(\alpha) = \sum_{t=0}^{N+1} f_0(y(t),\alpha,t) .$$

SC with respect to constant parameters α are called a gradient from $I(\alpha)$ on α: $(dI(\alpha)/d\alpha)^T \equiv \nabla_\alpha I(\alpha)$. SC are a coefficients of single-line relationship between the first variation of functional $\delta_\alpha I(\alpha)$ and the variations α of constant parameters $d\alpha$:

$$\delta_\alpha\, I(\alpha) = (\nabla_\alpha I(\alpha))^{\mathrm{T}} d\,\alpha = \frac{dI(\alpha)}{d\alpha} d\,\alpha \equiv \sum_{j=1}^{m} \frac{\partial I(\alpha)}{\partial \alpha_j} d\alpha_j .$$

The direct method of SC calculation inevitably requires a solution of cumbersome sensitivity equations to sensitivity functions $W(t)$. For instance, for functional $I(\alpha)$ we have following SC vector (row vector):

$$\frac{dI(\alpha)}{d\alpha} = \sum_{t=0}^{N+1} \left[\frac{\partial f_0(y(t),\alpha,t)}{\partial y(t)} W(t) + \frac{\partial f_0(y(t),\alpha,t)}{\partial \alpha} \right] .$$

$W(t)$ is the matrix of single-line relationship of the first variation of dynamic model output with parameter variations: $\delta y(t) = W(t) d\alpha$. For obtaining the matrix $W(t)$ it is necessary to decide bulky system equations – sensitivity equations. The *j*-th column of matrix $W(t)$ is made of the sensitivity functions $\partial y(t)/\partial \alpha_j$ with respect to component α_j of vector α. They satisfy a vector equation (if y is a vector) resulting from dynamic model (for y) by derivation on a parameter α_j.

For variable parameters such method essentially becomes complicated and practically is not applicable.

Application of the variation approach allows fundamentally to simplify process of calculation of SC concerning variable and constant parameters. To this problem it is devoted given paper for the dynamic systems described by difference equations with distributed memory on phase coordinates and variable parameters.

Variational method [6], ascending to Lagrange's, Hamilton's, Euler's memoirs, makes possible to simplify the process of determination of conjugate equations and formulas of account of SC. On the basis of this method it is an extension of quality functional by means of inclusion into it dynamic equations of object by means of Lagrange's multipliers and obtaining the first variation of extended functional on phase coordinates of object and on interesting parameters. Dynamic equations for Lagrange's multipliers are obtained due to set equal to a zero (in the first variation of extended functional) the functions before the variations of phase coordinates. Given simplification first variation of extended functional brings at presence in the right part only parameter variations, i.e. it is got the sensitivity functional according to concerning parameters.

In difference from other papers devoted to calculation of SC in given paper the generalized difference models are used. Thus variables and constant parameters enter into the right parts of difference equations of dynamic object, in an indicator of quality of system work, in the measuring device model and initial values of phase coordinates depend on constant parameters.

At the right part of the equations of object model there are also phase coordinates and variable parameters during the previous moments of time. Such discrete equations are similar numerical decisions of the integrated Volterra's equations.

It is proved that both methods to calculation of SC (either with use of Lagrange's functions or with use of sensitivity functions) yield the same result, but the first method it is essential more simple in the computing relation.

1. Problem statement

We suppose that the dynamic object is described by system of non-linear difference equations [13]

$$\mathrm{y}(t+1)=\sum_{s=0}^{t} K(t, y(s), \tilde{\alpha}(s), \alpha, s), \quad t=0, 1, 2, \ldots, N, \quad y(0)=y_0(\alpha). \tag{1}$$

Here: $\tilde{\alpha}(t), \alpha$ are a vector-columns of interesting variable and constant parameters; y is a vector-column of phase coordinates; $K(\cdot), y_0(\alpha)$ are known continuously differentiated limited vector-functions.

The quality of functioning of system it is characterised of functional

$$I(\tilde{\alpha}, \alpha)=\sum_{t=0}^{N} f_0(\mathrm{y}(t), \tilde{\alpha}(t), \alpha, t)+f_0(\mathrm{y}(N+1), \tilde{\alpha}(N+1), \alpha, N+1) \tag{2}$$

depending on $\tilde{\alpha}(t)$ and α. The conditions for function $f_0(\cdot), I(\cdot)$ are the same as for $K(\cdot)$. With use of a functional (2) the optimization problem (in the theory of optimal control) are named as the Bolts's problem. From it as the individual variants follow: Lagrange's problem (when there is only the first group of the summands) and Mayer's problem (when there is only the last summand – function from phase coordinates at a finishing point).

With the purpose of simplification of appropriate deductions with preservation of a generality in all transformations (1), (2) there are two vectors of parameters $\tilde{\alpha}(t), \alpha$. If in the equations (1), (2) parameters are different then it is possible formally to unit them in two vectors $\tilde{\alpha}(t), \alpha$, to use obtained outcomes and then to make appropriate simplifications, taking into account a structure of a vectors $\tilde{\alpha}(t), \alpha$.

Is shown also that the variation method without basic modifications allows to receive SC in relation to variable and constant parameters:

$$\delta I(\tilde{\alpha},\alpha)=\sum_{t=0}^{N+1}\frac{\partial I(\tilde{\alpha},\alpha)}{\partial\tilde{\alpha}(t)}\delta\tilde{\alpha}(t)+\frac{\partial I(\tilde{\alpha},\alpha)}{\partial\alpha}\delta\alpha\ . \tag{3}$$

$$\nabla_{\tilde{\alpha}(t)}I(\tilde{\alpha},\alpha)=\begin{pmatrix}\dfrac{\partial I(\tilde{\alpha},\alpha)}{\partial\tilde{\alpha}_1(t)} & \cdots & \dfrac{\partial I(\tilde{\alpha},\alpha)}{\partial\tilde{\alpha}_{m_1}(t)}\end{pmatrix}^{\mathrm{T}},\ t=0,1,2,\dots,N,N+1,$$

$$\nabla_{\alpha}I(\tilde{\alpha},\alpha)=\begin{pmatrix}\dfrac{\partial I(\tilde{\alpha},\alpha)}{\partial\alpha_1} & \cdots & \dfrac{\partial I(\tilde{\alpha},\alpha)}{\partial\alpha_{m_2}}\end{pmatrix}^{\mathrm{T}}.$$

By obtaining of results the obvious designations:

$$K(t,s)\equiv K(t,y(s),\tilde{\alpha}(s),\alpha,s),\ t=0,1,2,\dots,N;\ s=0,1,2,\dots t, \tag{4}$$

$$f_0(t)\equiv f_0(y(t),\tilde{\alpha}(t),\alpha,t),\ t=0,1,2,\dots,N+1$$

are used.

The indexes t,s in functions $K(t,y(s),\tilde{\alpha}(s),\alpha,s)$ and t in functions $f_0(y(t),\tilde{\alpha}(t),\alpha,t)$ also reflects not only obvious dependence on step number, but also that the kind of functions from a step to a step can change.

Let's receive the conjugate equations for calculation of Lagrange's multipliers and on the basis of them formulas for calculation of SC.

2. Conjugate equations

Complement a quality functional (2) by restrictions-equalities (1) by means of Lagrange's multipliers $\lambda(t),\ t=0,1,2,\dots,N+1$ (column vectors) and get the extended functional

$$I=I(\tilde{\alpha},\alpha)+\sum_{t=0}^{N}\lambda^{\mathrm{T}}(t+1)\left[-y(t+1)+\sum_{s=0}^{t}K(t,s)\right]+\lambda^{\mathrm{T}}(0)[-y(0)+y_0(\alpha)]= \tag{5}$$

$$=f_0(N+1)-\lambda^{\mathrm{T}}(N+1)y(N+1)+\sum_{t=0}^{N}\left[f_0(t)-\lambda^{\mathrm{T}}(t)y(t)\right]+\sum_{t=0}^{N}\sum_{s=0}^{t}\lambda^{\mathrm{T}}(t+1)K(t,s)+\lambda^{\mathrm{T}}(0)y_0(\alpha)\ .$$

Functional (5) complies with $I(\tilde{\alpha},\alpha)$ when (1) is fulfilled.

We consider equality

$$\sum_{t=0}^{N}\sum_{s=0}^{t}A(t,s)=\sum_{0}^{t=N}\sum_{t}^{s=N}A(s,t)\ . \tag{6}$$

For summand $\sum\limits_{t=0}^{N}\sum\limits_{s=0}^{t}\lambda^{\mathrm{T}}(t+1)K(t,s)$ in formulas (5) the equality (6) assumes the following form:

$$\sum_{t=0}^{N}\sum_{s=0}^{t}\lambda^{\mathrm{T}}(t+1)K(t,s)=\sum_{0}^{t=N}\sum_{t}^{s=N}\lambda^{\mathrm{T}}(s+1)K(s,t)\ .$$

Extended functional becomes now:

$$I=f_0(N+1)-\lambda^{\mathrm{T}}(N+1)y(N+1)+ \tag{7}$$

$$+\sum_{0}^{t=N}\left[f_0(t)-\lambda^{\mathrm{T}}(t)y(t)+\sum_{t}^{s=N}\lambda^{\mathrm{T}}(s+1)K(s,t)\right]+\lambda^{\mathrm{T}}(0)y_0(\alpha)\,.$$

We calculate the first variation of extended functional, caused by a variation of phase coordinates, and also a variation of variables and constant parameters:

$$\delta I=\sum_{t=0}^{N+1}\frac{\partial I}{\partial y(t)}\delta y(t)+\sum_{t=0}^{N+1}\frac{\partial I}{\partial\tilde{\alpha}(t)}\delta\tilde{\alpha}(t)+\frac{\partial I}{\partial\alpha}\delta\alpha. \tag{8}$$

The factors standing in the formula (8) before variations of phase coordinates look like:

$$\frac{\partial I}{\partial y(N+1)}=-\lambda^{\mathrm{T}}(N+1)+\frac{\partial f_0(N+1)}{\partial y(N+1)}, \tag{9}$$

$$\frac{\partial I}{\partial y(t)} = -\lambda^{\mathrm{T}}(t) + \sum_{t}^{s=N} \lambda^{\mathrm{T}}(s+1)\frac{\partial K(s,t)}{\partial y(t)} + \frac{\partial f_0(t)}{\partial y(t)},\ t = N, N-1, \ldots, 1, 0.$$

From equality to zero of these factors we receive the equations for Lagrange's multipliers:

$$\lambda^{\mathrm{T}}(N+1) = \frac{\partial f_0(N+1)}{\partial y(N+1)}, \tag{10}$$

$$\lambda^{\mathrm{T}}(t) = \sum_{t}^{s=N} \lambda^{\mathrm{T}}(s+1)\frac{\partial K(s,t)}{\partial y(t)} + \frac{\partial f_0(t)}{\partial y(t)},\ t = N, N-1, \ldots, 1, 0.$$

These equations are decided in the opposite direction changes of an independent integer variable t.

3. Sensitivity coefficients

In the equation (6) SC concerning variables and constant parameters look like:

$$\frac{\partial I}{\partial \tilde{\alpha}(N+1)} = \frac{\partial f_0(N+1)}{\partial \tilde{\alpha}(N+1)}, \tag{11}$$

$$\frac{\partial I}{\partial \tilde{\alpha}(t)} = \frac{\partial f_0(t)}{\partial \tilde{\alpha}(t)} + \sum_{t}^{s=N} \lambda^{\mathrm{T}}(s+1)\frac{\partial K(s,t)}{\partial \tilde{\alpha}(t)},\ t = N, N-1, \ldots, 1, 0,$$

$$\frac{\partial I}{\partial \alpha} = \frac{\partial f_0(N+1)}{\partial \alpha} + \sum_{0}^{t=N}\left[\frac{\partial f_0(t)}{\partial \alpha} + \sum_{t}^{s=N} \lambda^{\mathrm{T}}(s+1)\frac{\partial K(s,t)}{\partial \alpha}\right] + \lambda^{\mathrm{T}}(0)\frac{dy_0(\alpha)}{d\alpha}.$$

This result is more common in relation to appropriate results of monograph [13] and paper [16].

At reception SC (11) it was used extended functional (7).

4. Equivalence of sensitivity coefficient for initial (2) and extended (5) functionals

We take extended functional, presented in an initial part of the formula (5):

$$I = I(\tilde{\alpha}, \alpha) + \lambda^{\mathrm{T}}(0)[-y(0) + y_0(\alpha)] + \sum_{t=0}^{N} \lambda^{\mathrm{T}}(t+1)\left[-y(t+1) + \sum_{s=0}^{t} K(t,s)\right].$$

Before $\lambda^{\mathrm{T}}(\cdot)$ in square brackets there are the dynamic equations of the object which has been written down in the form of the equation of equality type. Hence, values of functions in square brackets are always equal to zero.

Let's calculate from both parts of the previous equation derivatives in the beginning on a vector of constant parameters α:

$$\frac{\partial I}{\partial \alpha} = \frac{\partial I(\tilde{\alpha}, \alpha)}{\partial \alpha} + \lambda^{\mathrm{T}}(0)\left[-W_{\alpha}(0) + \frac{dy_0(\alpha)}{d\alpha}\right] +$$

$$+ \sum_{t=0}^{N} \lambda^{\mathrm{T}}(t+1)\left[-W_{\alpha}(t+1) + \sum_{s=0}^{t}\left(\frac{\partial K(t,s)}{\partial y(s)}W_{\alpha}(s) + \frac{\partial K(t,s)}{\partial \alpha}\right)\right].$$

Before $\lambda^{\mathrm{T}}(\cdot)$ now there are sensitivity equations for a matrix of sensitivity functions. These equations are written down as in the form of restriction of equality type. Values of functions in square brackets also are always equal to zero.

Hence, SC rather both for initial functional and for its extended variant have identical values.

That the sensitivity equation had the specified appearance, it is necessary (1) to impose a condition of differentiability of $K(t,s)$ on phase coordinates and on considered parameters on the right member of equation of movement of dynamic object (1). On parameters α should be differentiated initial functions $y_0(\alpha)$.

We receive the same result and for SC in relation to variable parameters. The sensitivity equations of for each fixed value of argument of variable parameters $\tilde{\alpha}(j),\ j = 0, 1, \ldots, N+1$ have more complex form. They demand special consideration. Important that these sensitivity equations objectively exist.

5. Example

Let's receive the conjugate equations and SC for dynamic object described by system of non-linear ordinary difference equations [16]:

$$x(t+1) = f(x(t), \tilde{\alpha}(t), \bar{\alpha}, t), \;\; t = 0, 1, \ldots, N, \;\; x(0) = x_0(\bar{\alpha}). \qquad (12)$$

The dynamic equation (12) coincides with the equation (1) in the absence of the distributed memory, i.e. at $s = t$. Then it is necessary to replace $y(t)$ on $x(t)$ and

$$K(t,t) = f(t) \equiv f(x(t), \tilde{\alpha}(t), \bar{\alpha}, t), \; t = 0, 1, \ldots, N,$$

$$f_0(t) \equiv f_0(x(t), \tilde{\alpha}(t), \alpha, t), \;\; t = 0, 1, 2, \ldots, N+1.$$

The conjugate equations for Lagrange’s multipliers look like:

$$\lambda^{\mathrm{T}}(N+1) = \frac{\partial f_0(N+1)}{\partial x(N+1)},$$

$$\lambda^{\mathrm{T}}(t) = \lambda^{\mathrm{T}}(t+1)\frac{\partial f(t)}{\partial x(t)} + \frac{\partial f_0(t)}{\partial x(t)}, \; t = N, N-1, \ldots, 0.$$

From (11) it is received SC:

$$\frac{\partial I}{\partial \tilde{\alpha}(N+1)} = \frac{\partial f_0(N+1)}{\partial \tilde{\alpha}(N+1)},$$

$$\frac{\partial I}{\partial \tilde{\alpha}(t)} = \frac{\partial f_0(t)}{\partial \tilde{\alpha}(t)} + \lambda^{\mathrm{T}}(t+1)\frac{\partial f(t)}{\partial \tilde{\alpha}(t)}, \; t = N, N-1, \ldots, 1, 0,$$

$$\frac{\partial I}{\partial \alpha} = \frac{\partial f_0(N+1)}{\partial \alpha} + \sum_{t=0}^{N}\left[\frac{\partial f_0(t)}{\partial \alpha} + \lambda^{\mathrm{T}}(t+1)\frac{\partial f(t)}{\partial \alpha}\right] + \lambda^{\mathrm{T}}(0)\frac{dx_0(\alpha)}{d\alpha}.$$

6. The account of the measuring device model

At additional use of model of the measuring device it is necessary to make changes to problem statement:

$$f_0(t) \equiv f_0(\eta(t), \tilde{\alpha}(t), \alpha, t), \;\; t = 0, 1, 2, \ldots, N+1;$$

$$\eta(t) \equiv \eta(y(t), \tilde{\alpha}(t), \alpha, t), \;\; t = 0, 1, 2, \ldots, N+1.$$

In the received results it is necessary to execute small replacements:

$$\frac{\partial f_0(t)}{\partial y(t)} \text{ to replace on } \frac{\partial f_0(t)}{\partial \eta(t)}\frac{\partial \eta(t)}{\partial y(t)};$$

$$\frac{\partial f_0(t)}{\partial \tilde{\alpha}(t)} \text{ to replace on } \frac{\partial f_0(t)}{\partial \tilde{\alpha}(t)} + \frac{\partial f_0(t)}{\partial \eta(t)}\frac{\partial \eta(t)}{\partial \ddot{\alpha}(t)};$$

$$\frac{\partial f_0(t)}{\partial \alpha} \text{ to replace on } \frac{\partial f_0(t)}{\partial \alpha} + \frac{\partial f_0(t)}{\partial \eta(t)}\frac{\partial \eta(t)}{\partial \alpha}.$$

Conclusion

Variational method is applicability for calculation of SC of multivariate non-linear dynamic systems described by difference equations with the distributed memory on phase coordinates and variable parameters. Variables and constant parameters are present at object model, at model of the measuring device and at generalized quality functional for system (the Bolts's problem).

In a basis of calculation of SC the decision of the difference equations of object model in a forward direction of time and obtained difference equations for Lagrange's multipliers in the opposite direction of time. It is proved that both methods to calculation of SC (with use of Lagrange's functions or with use of sensitivity functions) yield the same result, but the first method it is essential more simple in the computing relation.

Results of present paper are applicable at design of high-precision systems and devices. This paper continues research in [13, 16].

It is possible generalization of the received results on the dynamic systems described by the difference equations, similar to the decision of ordinary integro-differential equations of Volterra's type. Such models include interconnected the ordinary difference equations and difference equations with the distributed memory in time on phase coordinates and variable parameters.

REFERENCES

1. Ostrovskij, G.M. & Volin, Yu.M. (1967) *Methods of Optimization of Chemical Reactors*. Moscow: Khimiya.
2. Rosenvasser, E.N. & Yusupov, R.M. (1969) *Sensitivity of Automatic Control Systems.* Leningrad: Energiya.
3. Krutyko P.D. (1969) The decision of a identification problem by a sensitivity theory method. *News of Sciences Academy of the USSR. Technical Cybernetics.* 6. pp. 146–153.
4. Petrov, B.N. & Krutyko, P.D. (1970) Application of the sensitivity theory in automatic control problems. *News of Sciences Academy of the USSR. Technical Cybernetics.* 2. pp. 202–212.
5. Gorodetsky, V.I., Zacharin, F.M., Rosenvasser, E.N. & Yusupov, R.M. (1971) *Methods of Sensitivity Theory in Automatic Control.* Leningrad: Energiya. 344 p.
6. Bryson, A.E. & Ho, Ju-Chi. (1972) *Applied Theory of Optimal Control*. Moscow: Mir.
7. Speedy, C.B., Brown, R.F. & Goodwin, G.C. (1973) *Control Theory: Identification and Optimal Control*. Moscow: Mir.
8. Gekher, K. (1973) *Theory of Sensitivity and Tolerances of Electronic Circuits*. Moscow: Sovetskoe radio.
9. Rouban, A.I. (1975) *Nonlinear Dynamic Objects Identification on the Base of Sensitivity Algorithm*. Tomsk: Tomsk State University.
10. Rosenvasser, E.N. & Yusupov, R.M. (eds) (1977) *Cybernetics Problems. Sensitivity Theory and its Application*. Vol. 23. Moscow: Svyaz.
11. Voronov, A.A. (1979) *Stability, Controllability, Observability*. Moscow: Nauka.
12. Rosenvasser, E.N. & Yusupov, R.M. (1981) *Sensitivity of Control Systems*. Moscow: Nauka.
13. Rouban, A.I. (1982) *Identification and Sensitivity of Complex Systems*. Tomsk: Tomsk State University.
14. Haug, E.J., Choi, K.K. & Komkov, V. (1988) *Design Sensitivity Analysis of Structural Systems*. Moscow: Mir.
15. Afanasyev, V.N., Kolmanovskiy, V.B. & Nosov, V.R. (1998) *The Mathematical Theory of Designing of Control Systems*. Moscow: Vysshaya shkola.
16. Rouban, A.I. (2021) The sensitivity coefficients for multivariate dynamic systems described by ordinary difference equations with variable and constant parameters. *Vestnik Tomskogo gosudarstvennogo universiteta. Upravlenie, vychislitel'naya tekhnika i informatika – Tomsk State University Journal of Control and Computer Science*. 55. pp. 65–70. DOI: 10.17223/19988605/55/8

Received: May 23, 2021

Rouban A.I. (2021) THE SENSITIVITY COEFFICIENTS FOR DYNAMIC SYSTEMS DESCRIBED BY DIFFERENCE EQUATIONS WITH THE DISTRIBUTED MEMORY ON PHASE COORDINATES AND VARIABLE PARAMETERS. *Vestnik Tomskogo gosudarstvennogo universiteta. Upravlenie, vychislitelnaja tehnika i informatika* [Tomsk State University Journal of Control and Computer Science]. 57. pp. 95–100

DOI: 10.17223/19988605/57/10

Рубан А.И. КОЭФФИЦИЕНТЫ ЧУВСТВИТЕЛЬНОСТИ ДЛЯ ДИНАМИЧЕСКИХ СИСТЕМ, ОПИСЫВАЕМЫХ РАЗНОСТНЫМИ УРАВНЕНИЯМИ С РАСПРЕДЕЛЕННОЙ ПАМЯТЬЮ ПО ФАЗОВЫМ КООРДИНАТАМ И ПЕРЕМЕННЫМ ПАРАМЕТРАМ. *Вестник Томского государственного университета. Управление, вычислительная техника и информатика.* 2021. № 57. С. 95–100

Вариационный метод применен для расчета коэффициентов чувствительности, которые связывают первую вариацию функционала качества работы системы (функционала Больца) с вариациями переменных и постоянных параметров, для многомерных нелинейных динамических систем, описываемых разностными уравнениями с распределенной памятью по фазовым координатам и переменным параметрам.

Ключевые слова: вариационный метод; коэффициент чувствительности; разностное уравнение; функционал качества работы системы; сопряженное уравнение; множитель Лагранжа.

ROUBAN Anatoly Ivanovich (Doktor of Technical Sciences, Professor of Computer Science Department of Institute of Space and Information Technologies, Siberian Federal University, Krasnoyarsk, Russian Federation).
E-mail: ai-rouban@mail.ru