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The sensitivity coefficients for dynamic systems described by nonlinear difference generalized equations with the distributed memory and characterized by generalized functionals

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Abstract. The variational method of calculation of sensitivity coefficients connecting first variation of quality functional with variations of variable and constant parameters for multivariate non-linear dynamic systems described by generalized difference equations with the distributed memory on phase coordinates and variable parameters is developed. The nonlinear quality functional also has a generalized form. Sensitivity coefficients are components of sensitivity functional and they are before variations of variable and constant parameters. The base of sensitivity coefficients calculation are the decision of generalized equations of the object model in the forward direction of discrete time and corresponding difference conjugate equations for Lagrange's multipliers in the opposite direction of discrete time.

Keywords: variational method; sensitivity coefficient; generalized difference equation; distributed memory; conjugate equation; Lagrange's multiplier

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Коэффициенты чувствительности для динамических систем, описываемых нелинейными разностными обобщенными уравнениями с распределенной памятью и характеризуемых обобщенными функционалами

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Аннотация. Вариационный метод применен для расчета коэффициентов чувствительности, которые связывают первую вариацию функционала качества работы системы с вариациями переменных и постоянных параметров, для многомерных нелинейных динамических систем, описываемых нелинейными разностными обобщенными уравнениями с распределенной памятью по фазовым координатам и переменным параметрам. Показатели качества работы систем также являются обобщенными нелинейными функционалами.

Ключевые слова: вариационный метод; коэффициент чувствительности; разностное уравнение; распределенная память; функционал качества работы системы; сопряженное уравнение; множитель Лагранжа

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Digital approaches to creation of new methods and algorithms of identification and control of dynamic systems lead to necessity of use discrete in time dynamic equations [1]. For dynamic systems the main problem at the analysis and syntheses methods of identification, control, optimization, stability [2–10] is the calculation of sensitivity functions and sensitivity coefficients (SC). The first-order sensitivity characteristics mostly are used. Later on we shall examine only SC of the first-order.

The SC are components of vector gradient from quality functional according to parameters and the sensitivity functional connects the first variation of quality functional with variations of variable and constant parameters.

Consider a vector output $y(t)$ of dynamic object model under discrete time $t \in [0,1,...,N+1]$ implicitly depending on vector constant α parameters and generalized $I(\alpha)$ functional constructed on a basis of $y(t)$ under $t \in [0,1,...,N+1]$ and on α parameters:

$$I(\alpha) = f_0(y(N+1), y(N), \ldots, y(1), y(0), \alpha) \equiv f_0(\cdot, \alpha).$$

SC with respect to constant α parameters are called a gradient from $I(\alpha)$ on α: $(dI(\alpha)/d\alpha)^T \equiv \nabla_\alpha I(\alpha)$. SC are a coefficients of single-line relationship between the first variation of functional $\delta_\alpha I(\alpha)$ and the variations $d\alpha$ of constant parameters α:

$$\delta_\alpha I(\alpha) = (\nabla_\alpha I(\alpha))^T d\alpha = \frac{dI(\alpha)}{d\alpha} d\alpha \equiv \sum_{j=1}^{m} \frac{\partial I(\alpha)}{\partial \alpha_j} d\alpha_j .$$

The direct method of SC calculation inevitably requires a solution of cumbersome sensitivity equations to sensitivity functions $W(t)$. For instance, for functional $I(\alpha)$ we have following SC vector (row vector):

$$\frac{dI(\alpha)}{d\alpha} = \sum_{t=0}^{N+1} \frac{\partial f_0(\cdot,\alpha)}{\partial y(t)} W(t) + \frac{\partial f_0(\cdot,\alpha)}{\partial \alpha} .$$

Here $W(t)$ is the matrix of single-line relationship of the first variation of dynamic model output with parameter variations: $\delta y(t) = W(t) d\alpha$. For obtaining the matrix $W(t)$ it is necessary to decide bulky system equations – sensitivity equations. The *j*-th column of matrix $W(t)$ is made of the sensitivity functions $\partial y(t)/\partial \alpha_j$ with respect to component α_j of vector α. They satisfy a vector equation (if y is a vector) resulting from dynamic model (for y) by derivation on a parameter α_j.

For variable parameters such method essentially becomes complicated and practically is not applicable.

Application of the variation approach allows fundamentally to simplify process of calculation of SC concerning variable and constant parameters. To this problem it is devoted given paper for the dynamic systems described by generalized difference equations with distributed memory on phase coordinates and variable parameters. The nonlinear quality functional also has a generalized form.

Variational method [6] makes possible to simplify the process of determination of conjugate equations and formulas of account of SC. On the basis of this method it is an extension of quality functional by means of inclusion into it dynamic equations of object with use of Lagrange’s multipliers and obtaining the first variation of extended functional on phase coordinates of object and on interesting parameters. Dynamic equations for Lagrange’s multipliers are obtained due to set equal to a zero (in the first variation of extended functional) the functions before the variations of phase coordinates. Given simplification first variation of extended functional brings at presence in the right part only parameter variations, i.e. it is got the sensitivity functional according to concerning parameters.

It is proved that both methods to calculation of SC (either with use of Lagrange's functions or with use of sensitivity functions) yield the same result, but the first method it is essential more simple in the computing relation.

1. Problem definition

We suppose that the dynamic object is described by system of generalized non-linear difference equations [1, 5]

$$y(t+1) = f(y(t), \dots, y(1), y(0); \tilde{\alpha}(t), \dots, \tilde{\alpha}(1), \tilde{\alpha}(0); \alpha, t), \quad t = 0, 1, \dots, N, \tag{1}$$
$$y(0) = y_0(\alpha).$$

Here: y is a vector-column of phase coordinates; $\tilde{\alpha}(t), \alpha$ are a vector-columns of interesting variable and constant parameters; $f(\cdot), y_0(\alpha)$ are known continuously differentiated limited nonlinear vector-functions.

The quality of functioning of system it is characterised of generalized nonlinear functional

$$I(\tilde{\alpha}, \alpha) = f_0(y(N+1), \dots, y(1), y(0); \tilde{\alpha}(N+1), \dots, \tilde{\alpha}(1), \tilde{\alpha}(0); \alpha), \tag{2}$$

depending from values of all phase coordinates and variable parameters: $y(t), \tilde{\alpha}(t), t = N+1, \dots, 1, 0,$ and from α. The conditions for function $f_0(\cdot)$ are the same as for functions $f(\cdot), y_0(\alpha)$.

With the purpose of simplification of appropriate deductions with preservation of a generality in all transformations (1), (2) there are two vectors of parameters $\tilde{\alpha}(t), \alpha$. If in the equations (1), (2) parameters are different then it is possible formally to unit them in two vectors $\tilde{\alpha}(t), \alpha$, to use obtained outcomes and then to make appropriate simplifications, taking into account a structure of a vectors $\tilde{\alpha}(t), \alpha$.

By obtaining of results the obvious designations:

$$f(t) \equiv f(y(t), \dots, y(1), y(0); \tilde{\alpha}(t), \dots, \tilde{\alpha}(1), \tilde{\alpha}(0); \alpha, t), \quad t = 0, 1, \dots, N, \tag{3}$$
$$f_0(\cdot) \equiv f_0(y(N+1), \dots, y(1), y(0); \tilde{\alpha}(N+1), \dots, \tilde{\alpha}(1), \tilde{\alpha}(0); \alpha),$$

are used.

In difference from other papers devoted to calculation of SC in given paper the generalized difference model (1) and generalized nonlinear functional (2) are used. In the previous papers of functions $f(t)$ and $f_0(\cdot)$ also were nonlinear, bat they contained the linear operator of summation, for example

$$f(t) \equiv \sum_{s=0}^{t} K(t, y(s), \tilde{\alpha}(s), \alpha, s), \quad t = 0, 1, 2, \dots, N, \quad y(0) = y_0(\alpha),$$

$$f_0(\cdot) \equiv \sum_{t=0}^{N+1} f_0(y(t), \tilde{\alpha}(t), \alpha, t) \quad .$$

The index t in functions $f(t)$ also reflects not only obvious dependence on step number, but also that the kind of functions from a step to a step can change.

Is shown also that the variation method allows to receive the SC in relation to variable and constant parameters:

$$\delta I(\tilde{\alpha}, \alpha) = \sum_{t=0}^{N+1} \frac{\partial I(\tilde{\alpha}, \alpha)}{\partial \tilde{\alpha}(t)} d\tilde{\alpha}(t) + \frac{\partial I(\tilde{\alpha}, \alpha)}{\partial \alpha} d\alpha. \tag{4}$$

$$\nabla_{\tilde{\alpha}(t)} I(\tilde{\alpha}, \alpha) = \left(\frac{\partial I(\tilde{\alpha}, \alpha)}{\partial \tilde{\alpha}_1(t)} \quad \cdots \quad \frac{\partial I(\tilde{\alpha}, \alpha)}{\partial \tilde{\alpha}_{m_1}(t)} \right)^{\mathrm{T}}, \quad t = 0, 1, 2, \dots, N, N+1,$$

$$\nabla_{\alpha} I(\tilde{\alpha}, \alpha) = \left(\frac{\partial I(\tilde{\alpha}, \alpha)}{\partial \alpha_1} \quad \cdots \quad \frac{\partial I(\tilde{\alpha}, \alpha)}{\partial \alpha_{m_2}} \right)^{\mathrm{T}}.$$

Let's receive the conjugate equations for calculation of Lagrange's multipliers and on the basis of them formulas for calculation of the SC.

2. Principal results

Complement a quality functional (2) by restrictions-equalities (1) by means of Lagrange's multipliers $\lambda(t), \ t = 0, 1, 2, \dots, N+1$ (column vectors) and get the extended functional

$$I = I(\tilde{\alpha}, \alpha) + \sum_{t=0}^{N} \lambda^{\mathrm{T}}(t+1)\left[-y(t+1) + f(t)\right] + \lambda^{\mathrm{T}}(0)[-y(0) + y_0(\alpha)]. \tag{5}$$

Functional (5) complies with $I(\tilde{\alpha}, \alpha)$ when (1) is fulfilled.

We calculate the first variation of extended functional, caused by a variation of phase coordinates and also a variation of variables and constant parameters:

$$\delta I = \delta_y I + \delta_{\tilde{\alpha}} I + \delta_\alpha I \equiv \sum_{t=0}^{N+1} \frac{\partial I}{\partial y(t)} dy(t) + \sum_{t=0}^{N+1} \frac{\partial I}{\partial \tilde{\alpha}(t)} d\tilde{\alpha}(t) + \frac{\partial I}{\partial \alpha} d\alpha. \quad (6)$$

Here:

$$\delta_y I = \left[\frac{\partial f_0(\cdot)}{\partial y(N+1)} - \lambda^{\mathrm{T}}(N+1) \right] dy(N+1) + \sum_{t=0}^{N} \left[\frac{\partial f_0(\cdot)}{\partial y(t)} - \lambda^{\mathrm{T}}(t) \right] dy(t) + \quad (7)$$

$$+ \sum_{t=0}^{N} \sum_{s=0}^{t} \lambda^{\mathrm{T}}(t+1) \frac{\partial f(t)}{\partial y(s)} dy(s).$$

We apply the equation

$$\sum_{t=0}^{N} \sum_{s=0}^{t} A(t,s) = \sum_{0}^{t=N} \sum_{t}^{s=N} A(s,t), \quad (8)$$

which has been received in [10] by calculation of SC for dinamice systems, described by more simple difference equations also with the distributed memory on phase coordinates and variable parameters. The proof of correctness of (8) equality is realized by an mathematical induction method.

For summand $\sum_{t=0}^{N} \sum_{s=0}^{t} \lambda^{\mathrm{T}}(t+1) \frac{\partial f(t)}{\partial y(s)} dy(s)$ in variation $\delta_y I$ the equality (8) has the following form:

$$\sum_{t=0}^{N} \sum_{s=0}^{t} \lambda^{\mathrm{T}}(t+1) \frac{\partial f(t)}{\partial y(s)} dy(s) = \sum_{0}^{t=N} \sum_{t}^{s=N} \lambda^{\mathrm{T}}(s+1) \frac{\partial f(s)}{\partial y(t)} dy(t). \quad (9)$$

Then

$$\delta_y I = \left[\frac{\partial f_0(\cdot)}{\partial y(N+1)} - \lambda^{\mathrm{T}}(N+1) \right] dy(N+1) + \quad (10)$$

$$+ \sum_{0}^{t=N} \left[\frac{\partial f_0(\cdot)}{\partial y(t)} - \lambda^{\mathrm{T}}(t) + \sum_{t}^{s=N} \lambda^{\mathrm{T}}(s+1) \frac{\partial f(s)}{\partial y(t)} \right] dy(t).$$

Derivatives standing in the formula (10) before variations of phase coordinates look like:

$$\frac{\partial I}{\partial y(N+1)} = -\lambda^{\mathrm{T}}(N+1) + \frac{\partial f_0(\cdot)}{\partial y(N+1)}, \quad (11)$$

$$\frac{\partial I}{\partial y(t)} = -\lambda^{\mathrm{T}}(t) + \sum_{t}^{s=N} \lambda^{\mathrm{T}}(s+1) \frac{\partial f(s)}{\partial y(t)} + \frac{\partial f_0(\cdot)}{\partial y(t)}, \; t = N, N-1, \ldots, 1, 0.$$

From equality to zero of these derivatives we receive the equations for Lagrange's multipliers:

$$\lambda^{\mathrm{T}}(N+1) = \frac{\partial f_0(\cdot)}{\partial y(N+1)}, \quad (12)$$

$$\lambda^{\mathrm{T}}(t) = \sum_{t}^{s=N} \lambda^{\mathrm{T}}(s+1) \frac{\partial f(s)}{\partial y(t)} + \frac{\partial f_0(\cdot)}{\partial y(t)}, \; t = N, N-1, \ldots, 1, 0.$$

These equations are decided in the opposite direction changes of an independent integer t variable.

Let's receive now formulas of SC calculation.

In the equations (in (6)) for $\delta_{\tilde{\alpha}} I$, $\delta_\alpha I$ the SC concerning variables and constant parameters look like:

$$\frac{\partial I}{\partial \tilde{\alpha}(N+1)} = \frac{\partial f_0(\cdot)}{\partial \tilde{\alpha}(N+1)}, \quad (13)$$

$$\frac{\partial I}{\partial \tilde{\alpha}(t)} = \frac{\partial f_0(\cdot)}{\partial \tilde{\alpha}(t)} + \sum_{t}^{s=N} \lambda^{\mathrm{T}}(s+1) \frac{\partial f(s)}{\partial \tilde{\alpha}(t)}, \; t = N, N-1, \ldots, 1, 0,$$

$$\frac{\partial I}{\partial \alpha} = \frac{\partial f_0(\cdot)}{\partial \alpha} + \sum_{0}^{t=N} \lambda^{T}(t+1) \frac{\partial f(t)}{\partial \alpha} + \lambda^{\mathrm{T}}(0) \frac{dy_0(\alpha)}{d\alpha}.$$

This result is more common in relation to appropriate results of monograph [5] and paper [10].

At reception SC (13) it was used the variations (6) of extended functional (5).

We prove equivalence of sensitivity coefficients for initial (2) and extended (5) functionals.

We take extended functional, presented in the formula (5). Before $\lambda^{\mathrm{T}}(\cdot)$ in brackets there are the dynamic equations of the object which has been written down in the form of the equations of equality type. Hence, values of functions in brackets are always equal to zero.

Let's calculate from both parts of the previous equation derivatives in the beginning on a vector of constant α parameters:

$$\frac{\partial I}{\partial \alpha} = \frac{\partial I(\tilde{\alpha}, \alpha)}{\partial \alpha} +$$

$$+\sum_{t=0}^{N} \lambda^{\mathrm{T}}(t+1)\left[-W_{\alpha}(t+1) + \sum_{s=0}^{t} \frac{\partial f(t)}{\partial y(s)} W_{\alpha}(s) + \frac{\partial f(t)}{\partial \alpha}\right] + \lambda^{\mathrm{T}}(0)\left[-W_{\alpha}(0) + \frac{d y_0(\alpha)}{d \alpha}\right].$$

Before $\lambda^{\mathrm{T}}(\cdot)$ now there are sensitivity equations for a matrix of sensitivity functions. These equations are written down as in the form of restrictions of equality type. Values of functions in brackets also are always equal to zero.

Hence, SC rather both for initial functional and for its extended variant have identical values.

For reception of the sensitivity equations it is necessary in equations (1) to impose a condition of differentiability for $f(t)$ on phase coordinates and on considered parameters. On α parameters should be differentiated initial functions $y_0(\alpha)$.

It is possible tu receive the same result and for SC on relation to variable parameters. The sensitivity equations for each fixed value of argument of variable parameters $\tilde{\alpha}(j),\ j = 0, 1, ..., N+1$ have more complex form. They demand special consideration. Important that such sensitivity equations are objectively exist.

Если при формиронии функционала качества работы системы используется дополнительный преобразователь фазовых координат, то необходимо учесть модель этого измерительного устройства

At additional use of model of the measuring device it is necessary to make changes to problem statement:

$$I(\tilde{\alpha}, \alpha) = f_0(\eta(N+1), \dots, \eta(1), \eta(0); \tilde{\alpha}(N+1), \dots, \tilde{\alpha}(1), \tilde{\alpha}(0); \alpha) \equiv f_0(\cdot);$$

$$\eta(t) \equiv \eta(y(t), \tilde{\alpha}(t), \alpha, t),\ t = 0, 1, 2, \dots, N+1.$$

In the received results it is necessary to execute small replacements:

$$\frac{\partial f_0(\cdot)}{\partial y(t)} \text{ to replace on } \frac{\partial f_0(\cdot)}{\partial \eta(t)} \frac{\partial \eta(t)}{\partial y(t)};$$

$$\frac{\partial f_0(\cdot)}{\partial \tilde{\alpha}(t)} \text{ to replace on } \frac{\partial f_0(\cdot)}{\partial \tilde{\alpha}(t)} + \frac{\partial f_0(\cdot)}{\partial \eta(t)} \frac{\partial \eta(t)}{\partial \tilde{\alpha}(t)};$$

$$\frac{\partial f_0(\cdot)}{\partial \alpha} \text{ to replace on } \frac{\partial f_0(\cdot)}{\partial \alpha} + \sum_{t=0}^{N+1} \frac{\partial f_0(\cdot)}{\partial \eta(t)} \frac{\partial \eta(t)}{\partial \alpha}.$$

3. Examples

Example 1. We consider that in dynamic system there is a memory on phase coordinates, but there is no memory on variable parameters, i.e.:

$$f(t) \equiv f(y(t), \dots, y(1), y(0); \tilde{\alpha}(t), \alpha, t),\ \ t = 0, 1, \dots, N. \tag{14}$$

Indicator of quality of system work is the same.

Structures of the conjugate equations (12) and SC (13) for constant parameters remain, but structure SC (13) for variable parameters becomes simpler:

$$\frac{\partial I}{\partial \tilde{\alpha}(t)} = \frac{\partial f_0(\cdot)}{\partial \tilde{\alpha}(t)} + \lambda^{\mathrm{T}}(t+1) \frac{\partial f(t)}{\partial \tilde{\alpha}(t)},\ t = N, N-1, \dots, 1, 0.$$

Example 2. In mathematical model of dynamic system there is no additional memory on phase coordinates, but there is a memory on variable parameters. At this variant

$$f(t) \equiv f(y(t); \tilde{\alpha}(t), \ldots, \tilde{\alpha}(1), \tilde{\alpha}(0); \alpha, t), \quad t = 0, 1, \ldots, N. \tag{15}$$

The indicator of a quality functional of system has the same appearance (2).

The structure of the conjugate equations (12) becomes simpler:

$$\lambda^{\mathrm{T}}(N+1) = \frac{\partial f_0(\cdot)}{\partial y(N+1)}, \quad \lambda^{\mathrm{T}}(t) = \lambda^{\mathrm{T}}(t+1)\frac{\partial f(t)}{\partial y(t)} + \frac{\partial f_0(\cdot)}{\partial y(t)}, \; t = N, N-1, \ldots, 1, 0.$$

The SC structures (13) for variables and constant parameters remain.

Example 3. In dynamic system there is no additional memory on phase coordinates and on variable parameters. Then

$$f(t) \equiv f(y(t); \tilde{\alpha}(t); \alpha, t), \quad t = 0, 1, \ldots, N, \tag{16}$$

but a quality functional of system has the same appearance (2).

In this variant the conjugate equations and SC to variable parameters become more simple:

$$\lambda^{\mathrm{T}}(N+1) = \frac{\partial f_0(\cdot)}{\partial y(N+1)}, \; \lambda^{\mathrm{T}}(t) = \lambda^{\mathrm{T}}(t+1)\frac{\partial f(t)}{\partial y(t)} + \frac{\partial f_0(\cdot)}{\partial y(t)}, \; t = N, N-1, \ldots, 1, 0;$$

$$\frac{\partial I}{\partial \tilde{\alpha}(t)} = \frac{\partial f_0(\cdot)}{\partial \tilde{\alpha}(t)} + \lambda^{\mathrm{T}}(t+1)\frac{\partial f(t)}{\partial \tilde{\alpha}(t)}, \; t = N, N-1, \ldots, 1, 0.$$

The SC to constant parameters remain former (13).

Conclusion

On the basis of a variation method the problem of effective calculation of SC for multivariate nonlinear dynamic systems described by the generalized difference equations with the distributed memory on phase coordinates and variable parameters is solved. The quality of functioning of systems it is characterized of generalized nonlinear functional (2) too. Variables and constant parameters are present at object model, at model of the measuring device and at generalized quality functional for system.

At the heart of SC calculation the decision of the difference equations of object model in a forward direction of time and obtained difference equations for Lagrange's multipliers in the opposite direction of time lies.

It is proved that both methods to calculation of SC (with use of Lagrange's functions or with use of sensitivity functions) yield the same result, but the first method it is essential more simple in the computing relation.

Results of present paper are applicable at design of high-precision systems and devices.

It is possible generalization of the received results on the dynamic systems described by. interconnected difference the ordinary equations and generalized equations with the distributed memory in time on phase coordinates and on variable parameters.

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