ВЕСТНИК ТОМСКОГО ГОСУДАРСТВЕННОГО УНИВЕРСИТЕТА

2025 Управление, вычислительная техника и информатика № 71

Tomsk State University Journal of Control and Computer Science

Original article
UDC 681.516
doi: 10.17223/19988605/71/8

Shortages of perishables control for a stochastic inventory system in retail through dynamic pricing

Anna V.[1] Kitaeva, Yu Cao[2]

[1, 2] *National Research Tomsk State University, Tomsk, Russian Federation*
[1] *kit1157@yandex.ru*
[2] *ch.cy@stud.tsu.ru*

Abstract. We address the management of a single product over a finite time horizon, with customer arrivals modeled as a heterogeneous compound Poisson process. The arrival rate depends on the inventory level and is adjusted by a time-dependent, unknown weight function, incorporating scenario of shortages. Retail price serves as the control variable for regulating this rate. Within the framework of diffusion approximation for the inventory process and a linear dependence of arrival intensity on price, we formulate and solve the expected revenue maximization problem concerning the weight function. Furthermore, two types of near-optimal weight functions are proposed, and the results of numerical optimization and simulation are presented.

Keywords: dynamic pricing; price sensitive demand; compound Poisson demand; diffusion approximation; shortages.

***For citation*:** Kitaeva, A.V., Yu, Cao (2025) Shortages of perishables control for a stochastic inventory system in retail through dynamic pricing. *Vestnik Tomskogo gosudarstvennogo universiteta. Upravlenie, vychislitelnaja tehnika i informatika – Tomsk State University Journal of Control and Computer Science*. 71. pp. 82–92. doi: 10.17223/19988605/71/8

Научная статья
doi: 10.17223/19988605/71/8

Контроль дефицита скоропортящихся товаров в стохастической системе управления запасами с использованием динамического ценообразования

Анна Владимировна Китаева[1], Юй Цао[2]

[1, 2] *Национальный исследовательский Томский государственный университет, Томск, Россия*
[1] *kit1157@yandex.ru*
[2] *ch.cy@stud.tsu.ru*

Аннотация. Рассматривается задача управления запасами единичного продукта на конечном временном горизонте с гетерогенным пуассоновским процессом спроса. Интенсивность спроса зависит от уровня запаса и весовой функции от времени, создающей возможность дефицита товара. Управляющим параметром выступает розничная цена. В рамках диффузионной аппроксимации уровня запасов и линейной зависимости интенсивности спроса от розничной цены решается задача максимизации средней прибыли относительно весовой функции и партии товара. Рассмотрено два вида весовых функций, приведены численные результаты оптимизации и имитационного моделирования.

Ключевые слова: динамическое ценообразование; чувствительный к цене спрос; составной пуассоновский спрос; диффузионная аппроксимация; дефицит.

***Для цитирования*:** Китаева А.В., Юй Цао. Контроль дефицита скоропортящихся товаров в стохастической системе управления запасами с использованием динамического ценообразования // Вестник Томского государственного университета. Управление, вычислительная техника и информатика. 2025. № 71. С. 82–92. doi: 10.17223/19988605/71/8

1. Introduction and problem statement

Research on inventory models for perishables dates back to the 1960s. A comprehensive review of fresh product supply chain management was provided in [1, 2], highlighting key developments in the field. Recent advancements in inventory theory have increasingly incorporated dynamic pricing mechanisms, with particular emphasis on demand-driven market environments [3].

In [4], we proposed a basic dynamic pricing control model, which allow us, triggering purchases, to sell all the perishable product at hand during its lifetime almost surely. In [5], we introduced a multiplicative coefficient into the basic model. In [6], we considered more complicated weight function introducing a power-law coefficient.

The presence of the adjustable coefficients in the weight functions allows us to use a linear approximation of the intensity-of-price dependence and optimize the zero-ending inventory retailing process. Moreover, the coefficients can help us also to solve the problem of fitting the demand rate in a real-life situation. This problem is the most challenging one in dynamic pricing, see, for example, [7].

In this paper, we do not specify the type of weight function, and we get the equation for the optimal weight function, which maximizes the expected revenue, and consider its approximate solution as one near-optimal weight function. This near-optimal weight function almost surely implies shortages depending on the range of the unknown constant it contains. Firstly, we consider the task of the expected revenue maximization with respect to the constant considering lost sales. Secondly, we modify the optimal weight function approximation so that the price at the beginning of the sales period becomes close to the base price and obtain the expected revenue in case of possible shortages.

Inventory shortages systems are generally classified into two categories: backlog systems and lost-sales systems [8]. In lost-sales systems, actual sales often underestimate true demand, resulting in inaccurate demand forecasts [9]. Additionally, lost sales can give rise to rationing and gaming behaviors, both of which contribute to the so-called bullwhip effect in supply chains. This effect causes instability, undermining operational efficiency across industries [10]. From both academic and practical perspectives, the assumption of lost sales is often considered more appropriate than backlogging [11]. In this study, we adapt a framework where unmet demand is treated as lost.

We begin by outlining the model's assumptions and notations. The supply chain under consideration consists of a single vendor and customers. Acting as a monopolist, the vendor aims to maximize revenue by procuring a fixed lot size Q_0 at a unit cost d and selling it over a predetermined period T. Inventory replenishment is not permitted within this fixed period.

The demand is assumed to be a compound Poisson process with intensity $\lambda(c(t))$, where $c = c(t)$ is a dynamic retail price per unit, the orders are independent identically distributed continuous random variables with the first and second moments a_1 and a_2 respectively.

The stochastic properties of the sales process and the expected revenue are determined using a diffusion approximation of the stock level process. This approach enables us to derive analytically manageable expressions. Thus, we assume that the stock level process satisfies the following equation:

$$dQ(t) = -a_1\lambda(c(t))dt + \sqrt{a_2\lambda(c(t))}dw(t),$$

where $w(\cdot)$ is the Wiener process.

2. Near-Optimal weight function for a large lot size

We consider the following model for controlling the intensity of customers' flow through dynamic pricing:

$$a_1\lambda\big(c(t)\big) = \frac{Q(t)}{T\varphi\big(t/T\big)}, \tag{1}$$

where $\varphi(\cdot)$ is an unknown weight function, $t \in [0, T]$. As stated in [12], "formulating problems in the framework of intensity control is considered a promising approach." We will call control model (1) as a general one.

The idea of such a model of intensity control as well as a lot of results in inventory control modelling belongs to Alexander Fedorovich Terpugov (1939-2009), former Head of the Department of Probability Theory and Mathematical Statistics at Tomsk State University, outstanding scientist and teacher.

Thus, the stock level process satisfies the following equation:

$$dQ(t) = -\frac{Q(t)}{T\varphi(t/T)}dt + \sqrt{\frac{a_2}{a_1}\frac{Q(t)}{T\varphi(t/T)}}dw(t).$$

2.1. The expected revenue and near-optimal weight function

It is easy to show that the expectation

$$E\{Q(t)\} = \bar{Q}(t) = Q_0 \exp\{-\psi(t/T)\}, \tag{2}$$

where $\psi(z) = \int_0^z \varphi^{-1}(x)dx.$

Denote $\overline{Q^2}(t) = E\{Q^2(t)\}$. Applying Ito's formula and averaging, we get

$$\frac{d\overline{Q^2}(t)}{dt} = -2\frac{\overline{Q^2}(t)}{T\varphi(t/T)} + \frac{a_2 Q_0 \exp\{-\psi(t/T)\}}{a_1 T\varphi(t/T)}$$

subject to $\overline{Q^2}(0) = Q_0^2$.

It follows that the variance

$$Var\{Q(t)\} = \frac{a_2 Q_0}{a_1}\exp\{-\psi(t/T)\}\left(1-\exp\{-\psi(t/T)\}\right). \tag{3}$$

Let us consider linear approximation of the intensity-of-price dependence

$$\lambda(c) = \lambda_0 - \lambda_1\frac{c(t)-c_0}{c_0}, \tag{4}$$

where c_0 is a stationary price corresponding stationary intensity λ_0 and parameter $\lambda_1 > 0$ characterizes the sensitivity of $\lambda(\cdot)$ to relative price‘s deviations from stationary price c_0.

Linear dependence of the customers' flow intensity on the price is common in literature, for example, in [13] the demand rate is supposed to be a linear function of the price.

From (1) and (4) we get

$$c(t) = c_0\left(1 + \frac{\lambda_0}{\lambda_1} - \frac{Q(t)}{a_1\lambda_1 T\varphi(t/T)}\right).$$

The average revenue at time unit

$$E\{c(t)a_1\lambda(c)\} = c_0 E\left\{\left(1+\frac{\lambda_0}{\lambda_1} - \frac{1}{a_1\lambda_1}\frac{Q(t)}{T\varphi(t/T)}\right)\frac{Q(t)}{T\varphi(t/T)}\right\} = c_0\left(1+\frac{\lambda_0}{\lambda_1}\right)\frac{\bar{Q}(t)}{T\varphi(t/T)} - \frac{c_0}{a_1\lambda_1}\frac{\overline{Q^2}(t)}{T^2\varphi(t/T)^2}.$$

The expected revenue over the cycle

$$\bar{S} = \int_0^T E\{a_1 c(t)\lambda(t)\}dt =$$

$$= \frac{c_0 Q_0}{\lambda_1}\left[(\lambda_0+\lambda_1)\int_0^1 e^{-\psi(z)}\psi'(z)dz - \frac{1}{a_1 T}\left(Q_0 - \frac{a_2}{a_1}\right)\int_0^1 e^{-2\psi(z)}\psi'^2(z)dz - \frac{a_2}{a_1^2 T}\int_0^1 e^{-\psi(z)}\psi'^2(z)dz\right].$$

Taking into account $\int_0^1 e^{-\psi(z)}\psi'(z)dz = 1 - e^{-\psi(1)}$, we get

$$\bar{S} = \frac{c_0 Q_0}{\lambda_1}\left[(\lambda_0+\lambda_1)\left(1-e^{-\psi(1)}\right) - \frac{1}{a_1 T}\left(Q_0 - \frac{a_2}{a_1}\right)\int_0^1 e^{-2\psi(z)}\psi'^2(z)dz - \frac{a_2}{a_1^2 T}\int_0^z e^{-\psi(z)}\psi'^2(z)dz\right].$$

Let us solve the task $-\int_0^1 \left[\left(\frac{a_1 Q_0}{a_2} - 1 \right) e^{-2\psi(z)} + e^{-\psi(z)} \right] \psi'^2(z) dz \quad \Rightarrow \max_{\psi(\cdot)}$ subject to $\psi(0) = 0$.

Following Euler's equation, optimal function $\psi(\cdot)$ satisfies

$$Ae^{-\psi}(\psi'' - \psi'^2) + \left(\psi'' - \frac{1}{2}\psi'^2 \right) = 0, \tag{5}$$

where coefficient $A = a_1 Q_0 / a_2 - 1$.

Coefficient $A \gg 1$, because Q_0 is usually large, so neglecting the last term in (5) we get $\psi'' - \psi'^2 = 0$. It follows that approximate solution of (5) has form $\varphi(z) = C - z$, where C is a constant.

If $C = 1$, then $E\{Q(T)\} = Var\{Q(T)\} = 0$, so the lot will be sold during time T almost surely. This case was considered in [4]. This is, so called, the basic zero-ending inventory dynamic price control model.

If $C > 1$, then the leftovers are possible almost surely. The expected stock level at the end of the period $\bar{Q}(T) = Q_0((C-1)/C)$.

If $C < 1$, then almost surely the vender will be out of the resources before the end of the period, that is, shortages are possible. Here, we consider this case.

2.2. The selling period's duration

Let us consider the Laplace transform of the probability density function of the stock level $\Phi(p,t) = E\{\exp(-pQ(t))\}$. By applying Ito's formula, we obtain

$$d\exp(-pQ(t)) = p\frac{Q(t)}{CT-t}\exp(-pQ(t))\left(1 + \frac{a_2}{2a_1}p\right)dt - p\exp(-pQ(t))\sqrt{\frac{a_2}{a_1}\frac{Q(t)}{CT-t}}dw(t). \tag{6}$$

After averaging (6), we get

$$(CT-t)\frac{\partial\Phi}{\partial t} + p\left(1 + \frac{a_2}{2a_1}p\right)\frac{\partial\Phi}{\partial p} = 0. \tag{7}$$

Solution of (7) has the form $\Phi(p,t) = \varphi\left(\frac{p(CT-t)}{p+\beta}\right)$, where $\varphi(\cdot)$ is an unknown function and parameter $\beta = 2a_1/a_2$. The density function $f(q,0) = \delta(q - Q_0)$, it follows $\Phi(p,0) = \varphi\left(\frac{pCT}{p+\beta}\right) = \exp(-pQ_0)$ or $\varphi(z) = \exp\left(-\frac{\beta z Q_0}{CT - z}\right)$. Finally, we get $\Phi(p,t) = \varphi\left(\frac{p(CT-t)}{p+\beta}\right) = \exp\left(-\frac{\beta p(CT-t)}{pt + \beta CT}Q_0\right)$.

By employing the inverse Laplace transform, we derive

$$f(q,t) = \exp\left(-\beta\frac{(CT-t)Q_0}{t}\right) \times \left[\delta(q) + \beta\exp\left(-\beta\frac{CTQ_0}{t}\right)\sqrt{\frac{CT(CT-t)Q_0}{t^2 q}}I_1\left(2\sqrt{\frac{CT(CT-t)}{t^2}\beta^2 Q_0 q}\right)\right]. \tag{8}$$

From (8) it follows that the cumulative distribution function of the length of time τ it takes to sell lot Q_0

$$F_\tau(t) = P(\tau \le t) = \exp\left(-\beta\frac{CT-t}{t}Q_0\right).$$

Thus, $P(\tau \le CT) = 1$, that is, the lot will be sold by the time CT almost surely.

The average of the selling period's duration $E\{\tau\} = \int_0^{CT}(1 - F_\tau(t))dt = CT\left(1 - \exp(\beta Q_0)\int_0^1 \exp\left(\frac{-\beta Q_0}{x}\right)dx\right)$.

Let us consider the case $\beta Q_0 >> 1$. In this case, we obtain

$$E\{\tau\} = CT\left(1 - \frac{1}{\beta Q_0} + o\left(\frac{1}{\beta Q_0}\right)\right). \tag{9}$$

2.3. The expected revenue taking shortages into account

Let us consider time point T_1, $T_1 = Tx_0$, where $x_0 = C\left(1 - 1/\beta Q_0\right)$, that is, taking the main part of (9) as the approximation of the average of the selling period's duration. In simulation, we take the moment when residual inventory fails to meet the requested purchase quantity of the terminal consumer.

In Figure 1 the results of illustrative simulation of the selling period's duration are presented. The thinning algorithm is used to generate a non-homogeneous Poisson process, and simulations conducted with 1000 iterations, and the average outcomes reported; 0,9 < C < 1, $T = 10$, Q_0 = 500, a_1 = 5, and $\lambda_0 / \lambda_1 = 4$, $\lambda_1 T = 100$; purchases are uniformly distributed over (0,10) or exponentially distributed. The black curve is the theoretical result, while the red curve represents the results of simulation in case of uniformly distributed purchases and the blue curve represents the results in case of exponentially distributed purchases.

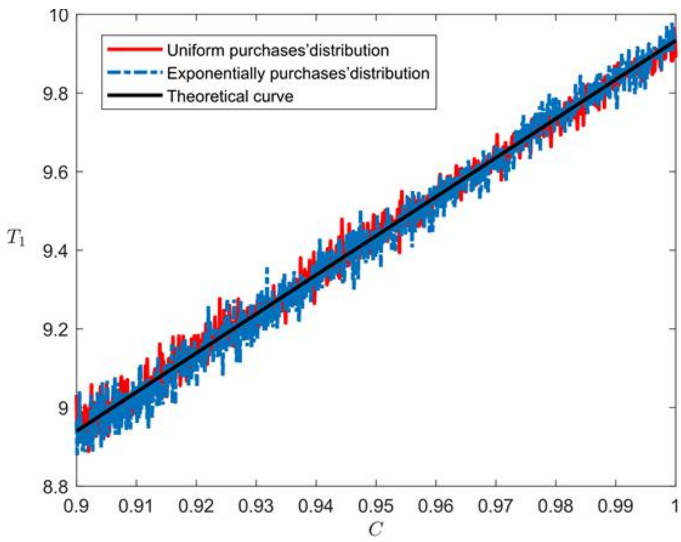

Fig. 1. T_1 simulation, T = 10, Q_0 = 500, purchases are Uniform (0,10) or Exp(5)

The parameter C significantly affects the duration of shortages. When C < 0.9 caution is advised, as the duration of shortages exceeds 10% of the entire period.

We consider the setting where unsatisfied demand is lost and assume that the demand intensity during shortages' period is static; $\lambda(c(T_1)) \approx \frac{a_2}{2a_1^2} \frac{1}{T(C - x_0)}$ and $c(T_1) \approx c_0\left(1 + \frac{\lambda_0}{\lambda_1} - \frac{a_2}{2a_1^2} \frac{1}{\lambda_1 T(C - x_0)}\right)$.

We define the average shortage penalty as follow

$$\bar{S}_s = -a_1 c(T_1)\lambda(c(T_1))(T - T_1) = -a_1 c_0\left(1 + \frac{\lambda_0}{\lambda_1} - \frac{a_2}{2a_1^2} \frac{1}{\lambda_1 T(C - x_0)}\right)\frac{1 - x_0}{C - x_0}.$$

Thus, we get

$$\bar{S}_{C<1} = \int_0^{T_1} E\{c(t)\lambda(t)\}dt + \bar{S}_s = c_0 Q_0\left[\left(1 + \frac{\lambda_0}{\lambda_1}\right)\frac{x_0}{C} - \frac{1}{C\lambda_1 T}\left(\left(\frac{Q_0 a_1 - a_2}{a_1^2}\right)\frac{x_0}{C} - \frac{a_2}{a_1^2}\ln\left(\frac{C - x_0}{C}\right)\right)\right] - \\ - a_1 c_0\left(1 + \frac{\lambda_0}{\lambda_1} - \frac{a_2}{2a_1^2} \frac{1}{\lambda_1 T(C - x_0)}\right)\frac{1 - x_0}{C - x_0}. \tag{10}$$

The expected revenue is monotonically increasing with respect to C, and weighted expected revenue $\frac{\bar{S}_{C<1}}{a_1 c_0}$ depends on four dimensionless system's parameters Q_0/a_1, λ_0/λ_1, $\lambda_1 T$, a_2/a_1^2, except C. In Figure 2 the results of illustrative simulation of weighted revenue's dependence on C are presented for different sets of the system's parameters. We apply the thinning algorithm to generate a non-homogeneous Poisson process, execute 1000 simulation iterations and take the average values. The black curves represent the theoretical results, and the red curves are the simulation results.

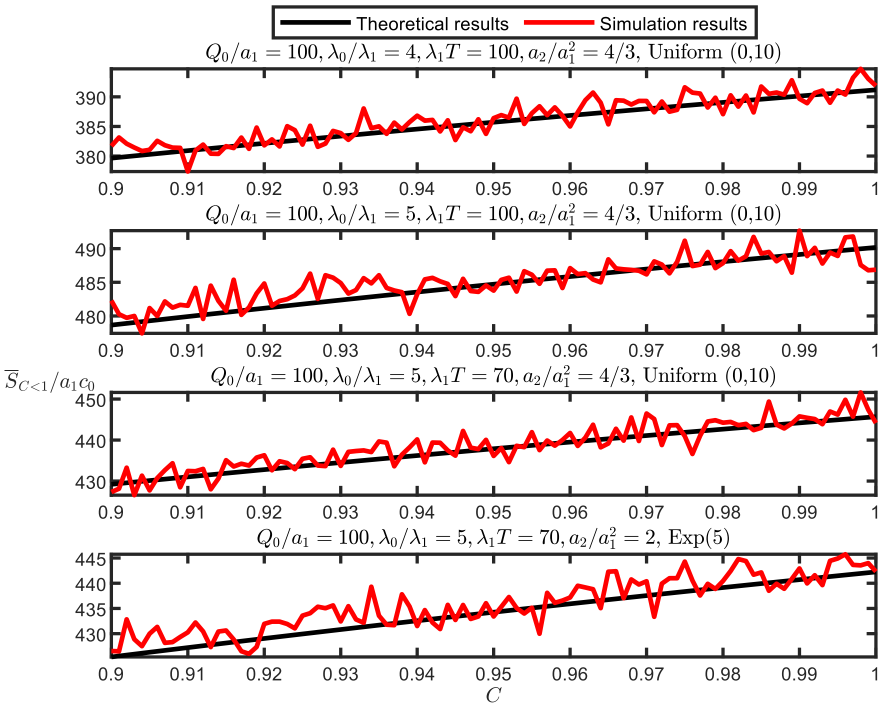

Fig. 2. $\bar{S}_{C<1}/a_1 c_0$ dependence on C for different sets of the system's parameters

Comparing the first three subplots in Figure 2 we see that the increasing ratio λ_0/λ_1 and $\lambda_1 T$ leads to an increase in price, thereby increasing the revenue significantly. Ratio a_2/a_1^2 determines the coefficient of variation of purchases and has no significant impact on revenue, compare the last two subplots in Figure 2.

There exists a monotonically increasing relationship between the weighted revenue $\bar{S}_{C<1}/a_1 c_0$ and parameter C, that is, as C tends to 1 from below, the duration of shortages decreases proportionally, thus enabling retailers to achieve higher revenue margins.

In Table numerical results of relative revenues calculation for the basic model $\bar{S}_{basic}/c_0 Q_0$ and the general one $\bar{S}_{C=1}/c_0 Q_0$ in case of $\lambda_0 T a_1 = Q_0$ are presented; $\bar{S}_{basic}/c_0 Q_0 \approx 1 - \frac{\lambda_0}{\lambda_1 \beta Q_0}$ and

$$\bar{S}_{C=1}/c_0 Q_0 \approx 1 + \frac{2\lambda_0\left(1 - 1/\beta Q_0 - \ln(\beta Q_0)\right)}{\lambda_1 \beta Q_0} - \frac{1}{\lambda_1 T}\left(\frac{\lambda_1}{\lambda_0} + 1 - \frac{1}{(\beta Q_0)^2}\right).$$

Revenues for the basic and general models

λ_0/λ_1	1	1	2	4	4
$\lambda_1 T$	400	400	200	100	100
βQ_0	2000/3	600	600	600	400
$\bar{S}_{basic}/c_0 Q_0$	0,9985	0,9983	0,9967	0,9933	0,9900
$\bar{S}_{C=1}/c_0 Q_0$	0,9790	0,9775	0,9600	0,9251	0,9034

The relative revenues for these two models are close to each other, the difference arises because of taking into account the lost sales.

The weighed revenue is a concave function with respect to a lot size. Figure 3 depicts weighted revenue $\overline{S}_{C<1} / a_1 c_0$ dependence on Q_0/a_1 for C = 0,9, 0,95 and 0,99; $\lambda_0 / \lambda_1 = 4,\ \lambda_1 T = 100,\ a_2 / a_1^2 = 4/3$.

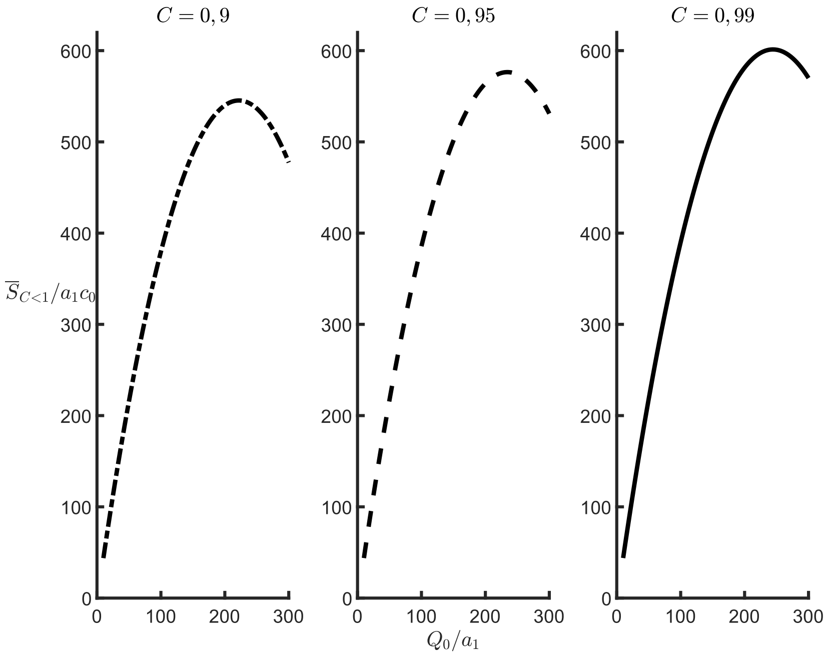

Fig. 3. $\overline{S}_{C<1} / a_1 c_0$ dependence on Q_0/a_1

As C tends to 1 from below, the optimal inventory level exhibits an upward trend, accompanied by a corresponding enhancement in the revenue.

3. Lot-weighted control model

Here, motivated by the form of the optimal weight function approximation, we are going to consider the following intensity of price dependence:

$$a_1 \lambda\left(c(t)\right) = \frac{CQ(t)}{CT - t}, \tag{11}$$

where $C > 0$ is a constant, $t < CT$. We will refer to this function as a lot-weighted one.

Stock level process is described by the following stochastic differential equation:

$$dQ(t) = -\frac{CQ(t)}{CT - t} dt + \sqrt{\frac{a_2}{a_1} \frac{CQ(t)}{CT - t}} dw(t). \tag{12}$$

Below we consider probabilistic characteristics of the stock level process and give the expression for the expected revenues for $C < 1$.

3.1. Probabilistic characteristics of the stock level process

Expectation and variance of $Q(t)$

$$E\left\{Q(t)\right\} = Q_0 g(t),\ \ Var\left\{Q(t)\right\} = \frac{a_2}{a_1} Q_0 g(t)\left(1 - g(t)\right),$$

where $g(t) = \left(1 - \dfrac{t}{CT}\right)^C,\ t \le CT.$

Consistent with the preceding context, we obtain the density function

$$f(q,t)=\exp\left(\frac{-\beta Q_0 g(t)}{1-g(t)}\right)\times\left(\delta(q)+\beta\exp\left(\frac{-\beta Q_0}{1-g(t)}\right)\frac{\sqrt{g(t)Q_0/q}}{1-g(t)}I_1\left(\frac{2\beta\sqrt{g(t)Q_0 q}}{1-g(t)}\right)\right).$$

It follows that the cumulative distribution function of τ

$$F_\tau(t)=P(\tau\le t)=\exp\left(-\beta Q_0\frac{g(t)}{1-g(t)}\right). \tag{13}$$

Note, that from (13) we get $P(\tau\le CT)=1$, that is, the shortages occur almost surely for $C < 1$. The average of the selling period's duration for $C < 1$

$$E\{\tau\}=\int_0^{CT}\left(1-F_\tau(t)\right)dt=CT\left(1-\int_0^1\exp\left(-\beta Q_0\frac{z^C}{1-z^C}\right)dz\right).$$

For $\beta Q_0 >> 1$

$$E\{\tau\}\approx CT\left(1-\int_0^1\exp\left(-\beta Q_0 z^C\right)dz\right)=CT\left(1-\frac{\Gamma(1+1/C)}{(\beta Q_0)^{1/C}}\right), \tag{14}$$

where $\Gamma(\cdot)$ is the gamma function.

3.2. The expected revenue taking shortages into account

Let us find the expected revenue for large lot size. Define the stationary (basic) price c_0 as a price satisfying equation $a_1\lambda(c_0)=Q_0/T$. Denote deviations from the stationary price $\Delta c(t)=c(t)-c_0$.

Using the Taylor expansion, we get $a_1\lambda(c(t))=CQ(t)/(CT-t)=a_1\lambda(c_0)+a_1\lambda'(c_0)\Delta c(t)+\ldots$ and $\Delta c(t)\approx\frac{1}{a_1\lambda'(c_0)}\left(\frac{CQ(t)}{CT-t}-\frac{Q_0}{T}\right)$.

Let us find conditional expectation $E\{Q(t)\,|\,Q(t)>0\}$, $E\{Q(t)\}=(1-\pi_0(t))E\{Q\,|\,Q(t)>0\}=Q_0 g(t)$, where $\pi_0(t)=P(\tau\le t)=\exp\left(-\beta Q_0\frac{g(t)}{1-g(t)}\right)$.

Consequently, $E\{Q(t)\,|\,Q(t)>0\}=\frac{Q_0 g(t)}{(1-\pi_0(t))}$, $E\left\{\frac{CQ}{CT-t}-\frac{Q_0}{T}\middle|Q(t)>0\right\}=\frac{CQ_0 g(t)}{(CT-t)(1-\pi_0(t))}-\frac{Q_0}{T}$,

$E\{\Delta c(t)|Q(t)>0\}=\frac{CQ_0 g(t)}{a_1\lambda'(c_0)(CT-t)(1-\pi_0(t))}-\frac{Q_0}{a_1\lambda'(c_0)T}$.

We take (14) as the approximation of the moment of shortages occurrence, $T_1 = Tx_0$, where $x_0=C\left(1-\frac{\Gamma(1+1/C)}{(\beta Q_0)^{1/C}}\right)$, and $Q(T_1)=\frac{Q_0\Gamma(1+1/C)}{(\beta Q_0)^{1/C}}$. We assume the setting where unsatisfied demand is lost and the demand intensity during shortages' period $T-T_1$ is static and defined by the intensity $\lambda(c(T_1))=\frac{CQ(T_1)}{a_1(CT-T_1)}=\frac{Q_0}{a_1T}\frac{C\Gamma(1+1/C)}{(C-x_0)(\beta Q_0)^{1/C}}=\frac{Q_0}{a_1T}=\lambda_0$; the retail price during the shortage period

$$c(T_1)=c_0+\Delta c(T_1)=c_0+\frac{1}{a_1\lambda'(c_0)}\left(\frac{CQ(T_1)}{CT-T_1}-\frac{Q_0}{T}\right)=$$
$$=c_0+\frac{1}{a_1\lambda'(c_0)}\frac{Q_0}{T}\left(\frac{C\Gamma(1+1/C)}{(C-x_0)(\beta Q_0)^{1/C}}-1\right)=c_0.$$

The expected revenue considering the lost sales at $[CT, T]$ for $\beta Q_0 >> 1$

$$\bar{S}_{C<1} = a_1 c_0 \lambda(c_0) \int_0^{CT} (1-\pi_0(t))dt + a_1 \left(\lambda(c_0) + c_0 \lambda'(c_0)\right) \int_0^{CT} (1-\pi_0(t)) \times$$
$$\times \left(\frac{CQ_0 g(t)}{a_1 \lambda'(c_0)(CT-t)(1-\pi_0(t))} - \frac{Q_0}{a_1 \lambda'(c_0) T} \right) dt - a_1 c_0 \lambda(c_0) T(1-x_0) = \quad (15)$$
$$= a_1 c_0 \lambda(c_0) T \left(1 + \frac{\lambda(c_0)}{c_0 \lambda'(c_0)} (1-x_0) \right) - a_1 c_0 \lambda(c_0) T (1-x_0).$$

Using linear approximation (4) and substituting $Q_0 / T = a_1 \lambda_0$, we can rewrite (15) as follows $\bar{S}_{C<1} = a_1 c_0 \lambda_0 T \left(1 - \frac{\lambda_0}{\lambda_1}(1-x_0) \right) - a_1 c_0 \lambda_0 T (1-x_0)$, or relative revenue

$$\bar{S}_{C<1} / c_0 Q_0 = C \left(1 - \frac{\Gamma(1+1/C)}{(\beta Q_0)^{1/C}} \right) \left(\frac{\lambda_0}{\lambda_1} + 1 \right) - \frac{\lambda_0}{\lambda_1}.$$

Relative revenue $\bar{S}_{C<1} / c_0 Q_0$ monotonically increases with respect to *C*. Tending *C* to one from below we get $\bar{S}_{C<1} / c_0 Q_0 \to 1 - \frac{\lambda_0}{\lambda_1 \beta Q_0} - \frac{1}{\beta Q_0}$. The result is the same as $\bar{S}_{basic} / c_0 Q_0$ taking into account that the last term appears due to the lost sales consideration.

Figure 4 depicts the relative revenues dependence on *C* for general (black lines) and lot-weighted (red lines) weight functions.

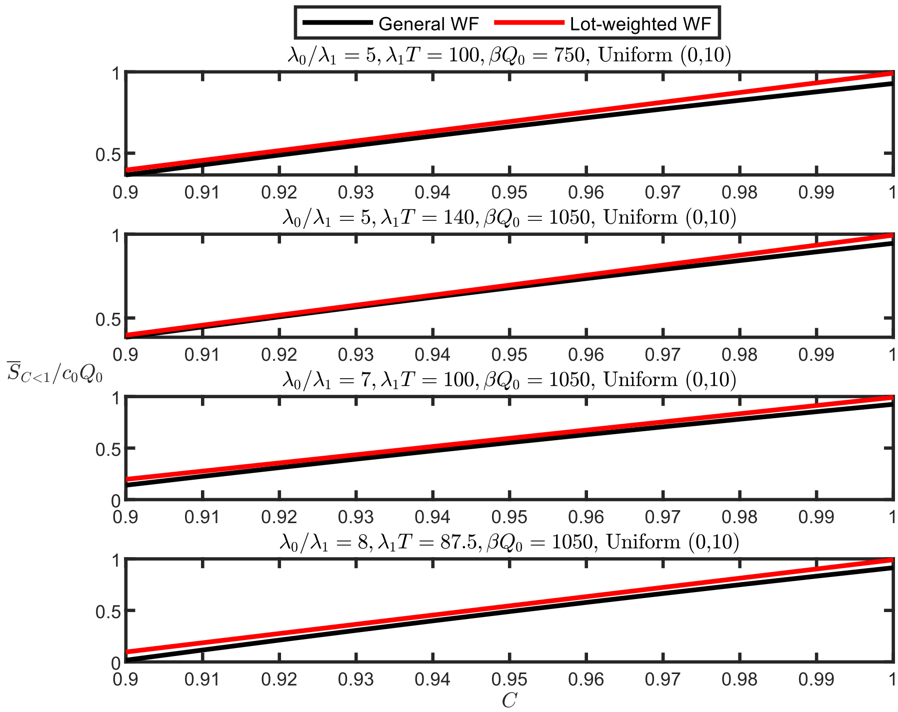

Fig. 4. Relative revenue $\bar{S}_{C<1} / c_0 Q_0$ dependence on *C*

Our analysis reveals a negligible revenue gap between the two models. The first two subplots in Figure 4 demonstrate that larger lot sizes mitigate this disparity. An increasing ratio λ_0 / λ_1 and decreasing $\lambda_1 T$ result in reduced revenue significantly, accompanied by a widening revenue gap between two models, compare the last three subplots in Figure 4. However, as *C* tends to 1 from below, the relative revenues of both models converge to 1, with the lot-weighted model consistently generating higher revenues.

Conclusion

This study investigates dynamic pricing for perishable products under a heterogeneous compound Poisson demand, employing a diffusion approximation framework of the stock level process. Such approximation works well for large lot sizes. Two near-optimal weight function models – general and lot-weighted – are proposed to optimize revenues addressing shortages by adjusting coefficient $C < 1$ in the weight functions.

The general model, derived analytically using the linear dependence of the intensity on price, provides closed-form expressions for the expected revenue and shortage duration, validated through simulation. It exhibits heightened sensitivity to C and depends on three dimensionless inventory system's parameters: βQ_0, λ_0 / λ_1, and $\lambda_1 T$. The modified, lot-weighted model, ensures more stable operation of the inventory system, depends only two parameters βQ_0, λ_0 / λ_1, and achieves a little higher revenue compared to the general model for C close to 1.

Numerical analyses demonstrate that both models enable the revenue optimization while balancing the shortages occurrence. Small values of the adjustable coefficient can lead to a significant reduction in revenue. These two control models can be used in case of the supply chain disruption during the sales period, which may lead to the need to shorten the period. Using these models allows us to sell all inventory by the new deadline and estimate the losses. The lot-weighed model has some advantages comparing with the general one: its relative revenue does not depend on $\lambda_1 T$, and at the beginning of the period we set basic price c_0 corresponding to basic intensity λ_0.

References

1. Krishna Manasvi, J. & Matai, R. (2022) Agri-fresh supply chain management: a systematic literature review. *Recent Advances in Industrial Production: Select Proceedings of ICEM*. pp. 449–457.
2. Shaabani, H. (2022) A literature review of the perishable inventory routing problem. *The Asian Journal of Shipping and Logistics*. 38(3). pp. 143–161. DOI: 10.1016/j.ajsl.2022.05.002
3. Yang, N. & Zhang, R. (2014) Dynamic pricing and inventory management under inventory-dependent demand. *Operations Research*. 62(5). pp. 1077–1094.
4. Kitaeva, A.V., Stepanova, N.V. & Zhukovskaya, A.O. (2019) Zero ending inventory dynamic pricing model under stochastic demand, fixed lifetime product, and fixed order quantity. *IFAC-PapersOnLine*. 52(3). pp. 2482–2487.
5. Kitaeva, A.V., Stepanova, N.V. & Zhukovskaya, A.O. (2020) Profit optimization for zero ending inventories dynamic pricing model under stochastic demand and fixed lifetime product. *IFAC-PapersOnLine*. 53(2). pp. 10505–10510.
6. Kitaeva, A.V., Stepanova, N.V. & Zhukovskiy, O.I. (2022) Profit Optimization with a Power-Law Adjustable Coefficient for Zero Ending Inventories Dynamic Pricing Model, Stochastic Demand, and Fixed Lifetime Product. *IFAC-PapersOnLine*. 55(10). pp. 1793–1797. DOI:10.1016/j.ifacol.2020.12.2796
7. Dolgui, A. & Proth, J.M. (2010) *Supply chain engineering: useful methods and techniques*. London: Springer.
8. Gayon, J.P., Massonnet, G., Rapine, C. & Stauffer, G. (2016) Constant approximation algorithms for the one warehouse multiple retailers problem with backlog or lost-sales. *European Journal of Operational Research*. 250(1). pp. 155–163.
9. Gruen, T.W., Corsten, D.S. & Bharadwaj, S. (2002) *Retail out-of-stocks: A worldwide examination of extent, causes and consumer responses*. Washington, DC: Grocery Manufacturers of America.
10. Hu, Q. (2019) Bullwhip effect in a supply chain model with multiple delivery delays. *Operations Research Letters*. 47(1). pp. 36–40.
11. Lim, A.E., Wei, Z.X. & Zhang, H. (2024) *Partial Backorder Inventory System: Asymptotic Optimality and Demand Learning*. arXiv preprint arXiv:2404.00046.
12. Gallego, G. & Van Ryzin, G. (1994) Optimal dynamic pricing of inventories with stochastic demand over finite horizons. *Management Science*. 40(8). pp. 999–1020.
13. Li, S., Zhang, J. & Tang, W. (2015) Joint dynamic pricing and inventory control policy for a stochastic inventory system with perishable products. *International Journal of Production Research*. 53(13). pp. 2937–2950.

Information about the authors:
Kitaeva Anna V. (Doctor of Physical and Mathematical Sciences, Professor of Institute of Applied Mathematics and Computer Science, National Research Tomsk State University, Tomsk, Russian Federation). E-mail: kit1157@yandex.ru
Yu Cao (Post-graduate Student, Institute of Applied Mathematics and Computer Science, National Research Tomsk State University, Tomsk, Russian Federation). E-mail: ch.cy@stud.tsu.ru

Contribution of the authors: the authors contributed equally to this article. The authors declare no conflicts of interests.

***Информация об авторах*:**
Китаева Анна Владимировна – доктор физико-математических наук, профессор Института прикладной математики и компьютерных наук Национального исследовательского Томского государственного университета (Томск, Россия). E-mail: kit1157@yandex.ru
Юй Цао – аспирант Института прикладной математики и компьютерных наук Национального исследовательского Томского государственного университета (Томск, Россия). E-mail: ch.cy@stud.tsu.ru

Вклад авторов: все авторы сделали эквивалентный вклад в подготовку публикации. Авторы заявляют об отсутствии конфликта интересов.

Received 17.02.2025; accepted for publication 02.06.2025

Поступила в редакцию 17.02.2025; принята к публикации 02.06.2025